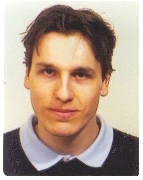
GLOBAL BIFURCATION ANALYSIS EXPLAINS OVEREXPLOITATION IN PREDATOR-PREY SYSTEMS WITH STRONG ALLEE EFFECT



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Introduction

Population models of ODEs mostly describe changes of abundances of species and nutrients in time. One way of analyzing these models is using bifurcation analysis. In parameter space different types of long-term behaviour can occur, namely stable equilibria, periodic behaviour, and chaos. Local bifurcations mark the transition boundaries in parameter space between regions that display these different types of behaviour, and can be found by following the individual equilibria and limit cycles, and determining their stability properties.

Some biologically relevant types of behaviour are not associated with the stability properties of equilibria and limit cycles. The boundaries of such regions in parameter space are marked by orbits that connect one or more equilibria and/or limit cycles. These are global bifurcations.

There are software programs, e.g. AUTO, that can numerically follow equilibria and periodic cycles, and indicate where local bifurcations occur. We develop extensions within AUTO to detect and follow connecting orbits. Here an example is presented of the detection and continuation of a point-to-point connecting orbit, that marks the boundary between coexistence in a prey-predator system with a strong Allee effect, and extinction of both the prey and predator populations due to overharvesting.

Example: overharvesting in an Allee-model

We introduce the dimensionless model by Bazykin-Berezovskaya (1998)

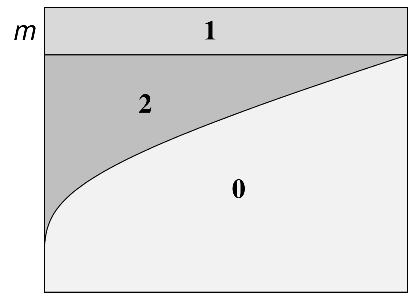
$$\frac{dx_1}{dt} = x_1(x_1 - l)(k - x_1) - x_1 x_2 \ ,$$
$$\frac{dx_2}{dt} = c(x_1 - m)x_2 \ ,$$

Table 1 ►

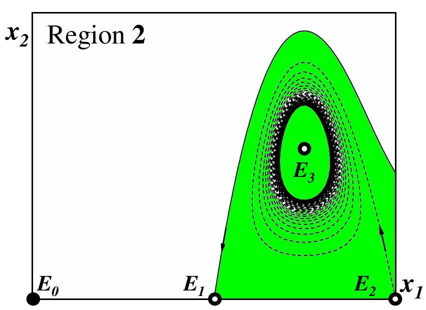
symbol	*meaning*	*value/range*
x_1/x_2	Prey/predator abundance	≥ 0
l	Prey Allee threshold	[0,1]
k	Carrying capacity	1
c	Biomass conversion ratio	1
m	Mortality rate of predator	[0,1]

The system has the equilibria
- Zero prey/predator abundance E_0
- Prey abundance at Allee threshold E_1
- Prey abundance at carrying capacity E_2
- Predator-prey coexistence E_3 (be it stable or periodic)

Two-parameter bifurcation diagram using AUTO97 (f77) and new extension ►

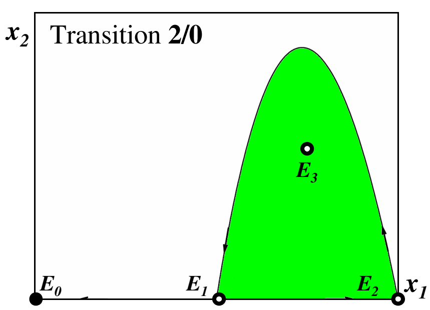


The resulting two-parameter bifurcation diagram shows three different regions:
Region **1**: Main attractor E_0 for $x_1 < k$, E_2 for $x_1 > k$ (= Allee effect). No predator population
Transition regions **1/2**: local bifurcation. Predator invasion criterion
Transition regions **2/0**: global bifurcation. See prey-predator phase plots below



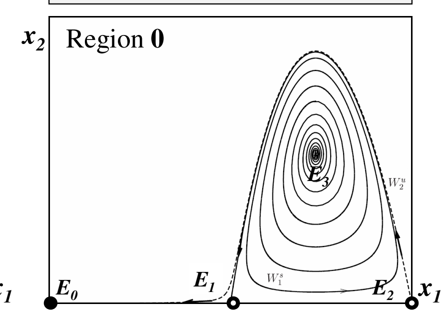
Coexistence

All initial values within the green zone converge to the system attractor E_3. Starting outside the green zone means extinction. Near transition curve regions **2/0** a limit cycle is born (Hopf bifurcation) and becomes attractor.



Heteroclinic cycle

At the transition between **2** and **0** the limit cycle connects with the two equilibria E_2 and E_1. Starting within the green zone stills means coexistence. The limit cycle is destroyed.



Overharvesting

For too low values of m, any initial predator invasion knocks the settled prey population out of E_2. The system instead converges to the attractor E_0. There is no longer coexistence possible; predator invasions kill the prey.

Discussion

Several biologically interesting and important phenomena, like overharvesting (as demonstrated here) and some transition boundaries where chaotic behaviour disappears, cannot be located and understood using local, but by using global bifurcation analysis. Here is demonstrated how a global bifurcation explains extinction of a predator-prey system with an Allee-effect. The technique used in the example above can be adapted for the detection and continuation of point-to-point, point-to-cycle and cycle-to-cycle connecting orbits in models of three and more dimensions. Its application will likely increase our understanding of the dynamics of ODE models of higher dimension.

References

G.A.K. van Voorn, L. Hemerik, M.P. Boer and B.W. Kooi. *Heteroclinic orbits indicate overexploitation in predator–prey systems with a strong Allee effect*. Math. Biosci. 209 (2007), 451-469
E.J. Doedel, B.W. Kooi, Yu.A. Kuznetsov and G.A.K. van Voorn. *Continuation of connecting orbits in 3D-ODEs: (I) point-to-cycle connections*. Int. J. Bif. Chaos, accepted