Detection and continuation of a heteroclinic point–to–cycle connection in the circuit model.

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Abstract

Accompanying manuscript to the demonstration of the detection and continuation of a heteroclinic point-to-cycle connection in the circuit model with the bifurcation software package AUTO, by use of the homotopy method described in the paper. The files are downloadable from http://www.bio.vu.nl/thb/research/project/globif.

1 Disclaimer

The following results have been obtained under Sun Solaris 8, using a FORTRAN *compiler f77 for* AUTO97*, and using a* FORTRAN *compiler f95 for* AUTO07P*. The results might differ slighthy using a different compiler or a different version of* AUTO*.*

2 Circuit model

The electronic circuit model of Freire et al. (1993) is discussed in the AUTO demos *tor* and *cir*. The equations are

$$\left\{ \begin{array}{rcl} r\dot{x} &=& -(\beta+\nu)x + \beta y - a_3x^3 + b_3(y-x)^3 \ , \\ \dot{y} &=& \beta x - (\beta+\gamma)y - z - b_3(y-x)^3 \ , \\ \dot{z} &=& y \ , \end{array} \right. \tag{1}$$

where $\gamma = 0$, $r = 0.6$, $a_3 = 0.328578$, $b_3 = 0.933578$, and ν and β are bifurcation parameters. With the use of HOMCONT it was demonstrated previously that a homoclinic connection to the origin occurs. Continuation in two-parameter dimension then leads to a Shil'nikov-Hopf bifurcation, occurring at

$$\nu = -1.026445, \beta = -2.330391 \cdot 10^{-5},$$

where a limit cycle bifurcates from the equilibrium, effectively turning the homoclinic connection into a heteroclinic one (see AUTO demo *cir*). The spurious results from the continuation in HOMCONT can be compared with the results from the application of the homotopy method described in the paper.

2.1 Local bifurcation analysis

A local bifurcation analysis starting from the trivial equilibrium $\nu = -1.5, \beta = -0.3$ and using β as a bifurcation parameter reveals a Hopf bifurcation point at $\beta = -4.551 \cdot 10^{-6}$. A two-parameter continuation then gives a limit point at $\beta = -2.0151819224, T = 8.8307075011$.

The command *make first* in the directory *01Cycle* gives

```
BR    PT  TY  LAB    PAR(2)
 1     1  EP    1  -3.00000E-01 ...
 1     3  HB    2  -4.55100E-06 ...
 1     7  BP    3   1.50000E+00 ...
 1    10  EP    4   4.50000E+00 ...

BR    PT  TY  LAB    PAR(2)
 2    10  EP    5   1.20924E+00 ...
```

```
BR    PT  TY  LAB    PAR(2)
 2    10  EP    6   1.20924E+00 ...
```

Restarting at label 2 *make second* gives

```
BR    PT  TY  LAB    PAR(2)             PERIOD       PAR(1)
 2     9  UZ    7  -3.20000E-01 ... 6.36461E+00  -1.50000E+00
 2    29  LP    8  -2.01518E+00 ... 8.83071E+00  -1.50000E+00
 2    30  EP    9  -2.00158E+00 ... 8.79666E+00  -1.50000E+00
```

In the directory *Cycle* the file *out.dat* provides the output.

2.2 Eigenfunction calculation

In the directory *02AdjEigFunc* the eigenfunction is obtained. First, the command *make compute* generates the file *compute*. Next, by typing *@compute C6* zeroes are added to the file *s.C6*. Also, a Floquet multiplier must be entered, for instance 1.1. The file *s.cir* is now generated.

The command *make first* generates

```
BR    PT  TY  LAB    PAR(11)            PAR(12)       PAR(13)
 1    19  BP    2   6.36461E+00 ...  4.20945E-08  0.00000E+00
 1    33  BP    3   6.36461E+00 ... -1.35793E+01  0.00000E+00
 1    50  EP    4   6.36461E+00 ... -3.05793E+01  0.00000E+00
```

Restarting at label 3 *make second* gives

```
BR    PT  TY  LAB    PAR(11)           PAR(12)       PAR(13)
 2   143  UZ    5   6.36461E+00 ... -1.35793E+01  9.99999E-01
 2   200  EP    6   6.36461E+00 ... -1.35793E+01  1.54119E+00
```

The eigenfunctiondata is taken from the cycle with a period of $T = 6.3646138318$, while the log multiplier $\lambda = -13.579343187$.

2.3 Connection

To obtain an initial approximation of the connecting orbit we use the base point

$$\Psi[x^+] = x_2^+(0) - 0.46460309314$$

in the program CONTENT. After this a time-integrated connecting orbit, starting from

$$x_1 = -9.941363 \cdot 10^{-5}\ ,\ x_2 = -1.028759 \cdot 10^{-5}\ ,\ x_3 = -3.330909 \cdot 10^{-6}$$

is obtained with MATLAB. The connection data are stored in the file *CEY.dat*.

2.4 Homotopy step 1

The approximate connecting orbit can be improved using the homotopy method, as discussed in the paper. With *make first* in the directory *03HomotopyH1* the connection is improved by varying the connection time $PAR(13)$ and searching the point $h_1 = 0$, where h_1 is one of the two homotopy parameters

```
BR    PT  TY  LAB    PAR(17)          PAR(13)
 1    11  UZ    3   1.32078E-06 ... 1.15982E+01 ...
 1    20  EP    4  -2.46728E-01 ... 1.09911E+01 ...
```

The uzer-defined data is exported.

2.5 Homotopy step 2

The approximate connecting orbit is further improved in the directory *03HomotopyH2*. The command *make first* gives

```
BR    PT  TY  LAB     PAR(1)            PAR(18)
 1    10        2  -1.51505E+00 ...  3.50187E-01 ...
 1    17  UZ    3  -1.50050E+00 ... -1.48415E-07 ...
 1    20  EP    4  -1.48466E+00 ... -2.44501E-01 ...
```

where $PAR(18)$ is the second homotopy parameter, which is zero at the uzer-defined point.

The relevant data is exported.

2.6 Continuation

The connecting orbit can be improved further by increasing the connection time in the directory *05P2Ccont* with *make first*

```
BR    PT  TY  LAB    PAR(13)           PAR(1)
 1    10        2   1.16526E+01 ... -1.50050E+00 ...
 1    20        3   1.20810E+01 ... -1.50050E+00 ...
 1    30        4   1.29782E+01 ... -1.50050E+00 ...
 1    40        5   1.56611E+01 ... -1.50050E+00 ...
 1    50        6   1.85159E+01 ... -1.50050E+00 ...
 1    57  UZ    7   2.00000E+01 ... -1.50050E+00 ...
 1    60  EP    8   2.21815E+01 ... -1.50050E+00 ...
```

Restarting from the uzer-defined point we continue the connecting orbit in a two-dimensional parameter space (ν, β). The forward continuation *make second*

```
BR    PT  TY  LAB    PAR(1)            PAR(2)
 1    25        9  -1.64769E+00 ... -3.99247E-01 ...
 1    50       10  -1.89633E+00 ... -5.17901E-01 ...
```

1	75		11	-2.12154E+00	...	-6.10928E-01	...
1	100		12	-2.31735E+00	...	-6.82455E-01	...
1	125		13	-2.49984E+00	...	-7.42796E-01	...
1	150		14	-2.67830E+00	...	-7.97266E-01	...
1	175		15	-2.89007E+00	...	-8.57774E-01	...
1	188	UZ	16	-3.00000E+00	...	-8.87980E-01	...
1	200		17	-3.10298E+00	...	-9.15778E-01	...
1	225		18	-3.30647E+00	...	-9.69850E-01	...
1	250		19	-3.46426E+00	...	-1.01142E+00	...
1	275		20	-3.56144E+00	...	-1.03699E+00	...
1	300	EP	21	-3.62778E+00	...	-1.05447E+00	...

The backward continuation *make third*

BR	PT	TY	LAB	PAR(1)		PAR(2)	
1	25		9	-1.34350E+00	...	-2.26956E-01	...
1	50		10	-1.02662E+00	...	-1.21859E-04	...
1	51	LP	11	-1.02648E+00	...	-2.01466E-10	...
1	75		12	-1.34574E+00	...	-2.28355E-01	...
1	100		13	-1.62181E+00	...	-3.85827E-01	...
1	125		14	-1.87119E+00	...	-5.06704E-01	...
1	150		15	-2.10879E+00	...	-6.05987E-01	...
1	175		16	-2.33949E+00	...	-6.90071E-01	...
1	199	LP	17	-2.53414E+00	...	-7.53574E-01	...
1	200		18	-2.53245E+00	...	-7.53046E-01	...
1	212	LP	19	-2.49042E+00	...	-7.39807E-01	...
1	225		20	-2.53704E+00	...	-7.54477E-01	...
1	250		21	-2.73162E+00	...	-8.12859E-01	...
1	275		22	-2.95353E+00	...	-8.75289E-01	...
1	300		23	-3.17372E+00	...	-9.34675E-01	...
1	307	LP	24	-3.22015E+00	...	-9.47013E-01	...
1	325		25	-3.09119E+00	...	-9.12617E-01	...
1	345	LP	26	-2.97311E+00	...	-8.80651E-01	...
1	350		27	-2.98871E+00	...	-8.84905E-01	...
1	375		28	-3.17060E+00	...	-9.33843E-01	...
1	400	EP	29	-3.36654E+00	...	-9.85692E-01	...

Observe that the backward continuation detects a limit point, which coincides with the Shil'nikov-Hopf bifurcation point.

Figure 1 displays a point-to-cycle connection in a x_1,x_2-plot at some selected parameter values. Figure 2 shows the results of the two-parameter continuation in HOMCONT and our continuation of the heteroclinic connection. Label 5 is the starting point of the continuation of the homoclinic connection, that terminates at label 1. Beyond label 1, HOMCONT gives spurious results. However, label 1 coincides with label 9, which indicates the bouncing point

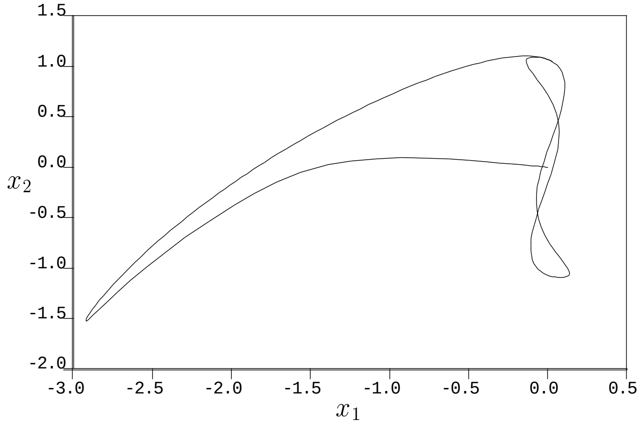

Figure 1: A point-to-cycle connection depicted in x_1 and x_2 for the electronic circuit model.

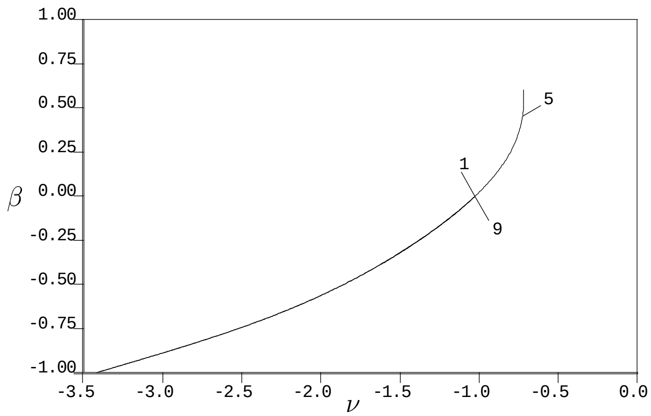

Figure 2: Continuation of the point-to-cycle connection in two-parameter space (ν, β). See text for further explanation.

of the continuation curve of the heteroclinic connection at the point where the Shil'nikov-Hopf bifurcation occurs.

3 Acknowledgments

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References

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