An Approach to the Study of Ancient Archery using Mathematical Modelling[1]

B.W. Kooi and C.A. Bergman

Abstract

The archer's bow is a machine whose purpose is to impart the stored energy effectively and accurately to propel the arrow. A Mathematical modelling of different bow types shows how their engineering characteristics define their performances.

[1] *Antiquity* **71**:(271) 124–134 (1997)

1 Introduction

One way of studying ancient bows is to make replicas and use them for experiments (Pope 1974; Miller *et al.* 1986; McEwen *et al.* 1991). In the present paper, the emphasis is on a different approach, the use of mathematical models which permit theoretical experiments using computers to gain insight into the performance of different types of bows. The use of physical laws and measured quantities, such as the specific mass of materials, in constitutive relations yields mathematical equations. In many cases, the complexity of the models obtained does not permit the derivation of the solutions by paper and pencil. Computers can then be used to approximate the solution; however, even this procedure will often necessitate simplifications. Essential detailed information may be missing, or assumptions need to be made to keep the model manageable. In that case, the model has to be validated by comparison of predicted results with actual measured quantities to justify the assumptions; fortunately, replicas can be employed for this purpose.

Mathematical models must accommodate all factors that determine the action of the bow; these are the “design parameters”. Calculations are possible only if all the parameters are known. Descriptions of bows in the literature are often incomplete, so that comprehensive evaluation becomes impossible.

Theoretical experiments with models deal to a large extent with the influence of the design parameters on the performance of the bow. This presupposes a definition of good performance that fits the context of interest. Flight shooters are only interested in a large initial velocity, whereas target archers want a bow to shoot smoothly and accurately.

In the 1930s, bows and arrows became the object of study by scientists and engineers (Hickman *et al.* 1947; Klopsteg 1987). They performed experiments to determine the influence of different design parameters. Hickman made a very simple mathematical model for flatbows. Their work had a major impact on the design and construction of modern bows and arrows.

In the next section, the principles behind Kooi's mathematical model are highlighted (a full description of the model is beyond the scope of this paper; the reader is referred to Kooi & Sparenberg 1980; Kooi 1981, 1983, 1991, 1994). The model developed is much more advanced, so that more detailed information is obtained; it gives a better understanding of the action of rather general types of bow, including both ancient and modern examples.

This model was validated by a comparison of the measured initial velocity of an arrow shot with a modern bow with the predicted value (Tuijn & Kooi 1992). As part of the *Mary Rose* project (Hardy 1992), the measured weight of a medieval longbow replica was correlated with the predicted value. In both cases the predicted values matched actual measurements.

The aim of the present paper is to use the model to evaluate the performance of bows past and present. The function of the siyahs or ‘ears’ of the Asiatic composite bow is examined, along with the reasons for the differences in the performance of the longbow and the Turkish bow in flight shooting.

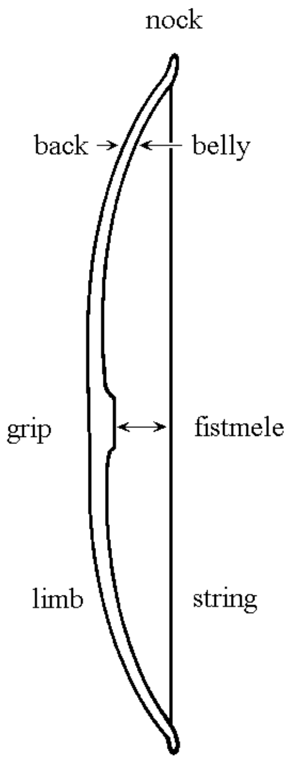


Figure 1: Anatomy of the bow.

2 Mathematical modelling

In general, the bow proper consists of two equal elastic limbs, often separated by a rigid middle part, the grip. Not all design types follow this idealized description. The *c.* 2 m long Japanese bow, with the grip positioned at a point approximately one-third along its length measured from the lower nock, bends with an asymmetrical profile. Such design variations must always be considered before applying a mathematical model to the study of bow function and performance. In this study, the model describes bows which bend symmetrically.

The bow is braced by fastening a string between both ends of the limbs (Figure 1). After an arrow is set on the string, the archer pulls the bow from the braced position into full draw storing potential energy in the elastic parts of the bow. After aiming, the arrow is released. The force in the string accelerates the arrow and transfers part of the available energy as kinetic energy to the arrow; with the bow held in its place, the archer feels a recoil force in the bowhand. After the arrow has left the string, the bow returns to the braced position because of damping.

2.1 Hickman's model

Hickman developed an analytical method to determine the dynamic forces, the accelerations and velocities of the arrow, string and bow limbs (Hickman *et al.* 1947). His model, though very simple, reveals some characteristics of the bow and arrow. He replaced the flexible limbs by rigid ones, connected to the grip by linear elastic hinges (Figure 2). The mass of the rigid limbs is concentrated at the tips where the string is connected to the

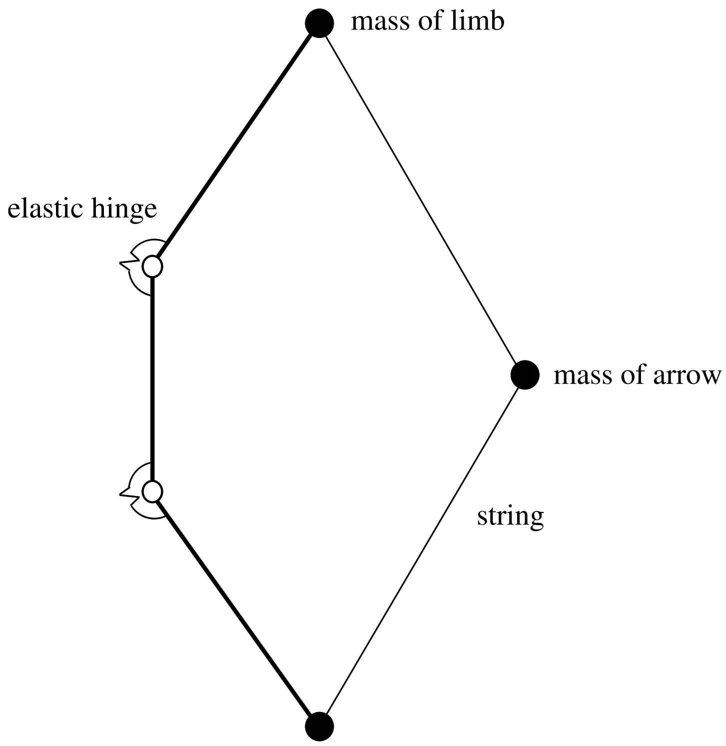


Figure 2: Hickman's model of a bow. Two linear elastic hinges and rigid limbs with point mass at the tip.

limbs. Hickman assumed that the cross-section of the limb is rectangular, that the thickness is uniform along the limb, and that the width diminishes gradually towards the tip. The location and strength of the elastic hinges and the masses at the tips are determined so that the essence of the mechanical behavior of the limbs will be alike both in this model and a model in which the limbs are treated as slender elastic beams for small deflections. The string is assumed to be inextensible and its mass is accounted for by adding one-third of the string mass (designated the added mass) to the mass of the arrow. The additional two-thirds are added to the mass at the tip of each limb. After these manipulations, the string can be considered inextensible and massless.

Under general conditions, all the additional energy stored in the elastic hinges by drawing this bow from the braced position into a fully drawn position is transferred to the arrow plus the added mass, both of which possess the same speed at arrow release.

In the Hickman model, after the arrow is released, the limbs and string, with the concentrated added mass in the middle, oscillate around the braced position. At this stage the total energy in hinges, limbs, and string equals the potential energy in the hinges in the braced position plus the kinetic energy of the added mass of arrow release.

This simple model shows a characteristic feature of the bow while being shot. Upon release, the arrow is accelerated and the limbs absorb energy as kinetic energy. Immediately before the string returns to the braced position, the limbs are then decelerated by a rather large force in the string. In this way, energy is transferred by the string to the arrow and added mass. This makes the bow a rather efficient mechanism.

Further, we observe the importance of the lightness of the string: a smaller added mass means more kinetic energy in the arrow. If the mass of the string were zero, all additional energy accumulated in the bow during the draw would be imparted to the arrow.

2.2 Our model

In the present mathematical model the limbs are considered as beams that store elastic energy by bending: the fibres in the limbs at the back side (Figure 1) are elongated and those on the belly side are compressed. The Bernoulli-Euler equation describes the change in the curvature of the beam as a function of the "bending moment". Both functions depend on the position along the limb. The proportionality constant is "flexural rigidity". This bending moment originates from the force in the string. When the bow is drawn by the archer, the force exerted at the middle of the string causes an increase in the bending of the limbs.

After the archer releases the arrow, the motion of the limbs, string and arrow is mathematically described by Newton's law of motion. The mass distribution along the bow is part of the governing equation for the limb besides the flexural rigidity distribution.

In a model that resembles the one previously developed by Kooi (1991), the limbs are replaced by rigid elements (Figure 3) endowed with point masses and connected by elastic hinges. The distributed mass of each element is concentrated at the middle point, while the distributed flexural rigidity is concentrated into the elastic hinge. Newton's law of motion for the point masses forms the governing equations. For non-recurve bows, the string is modelled in a similar way, with springs and point masses. In the case of the static-recurve and working-recurve bows, only two equal springs are used, and the mass of the string is equally divided between the arrow and both tips of the limbs.

In the present model, the governing equations were obtained following a slightly different route. Formulating the beam equation of the limb first, Kooi discretized these mathematically, instead of using the physical discretization described above. Mathematical discretization has advantages from a numerical analysis point of view. In this way formulations are obtained that can be implemented rather easily into a computer code. Computer simulations can then give the calculated motion of the arrow during its release.

2.3 Design parameters

The important quantities in the model which determine the mechanical action of the bow are:

bow:	length of the limbs
	length of the grip
	shape of the unstrung limbs
	shape of cross-section of the limbs at all positions along the limbs

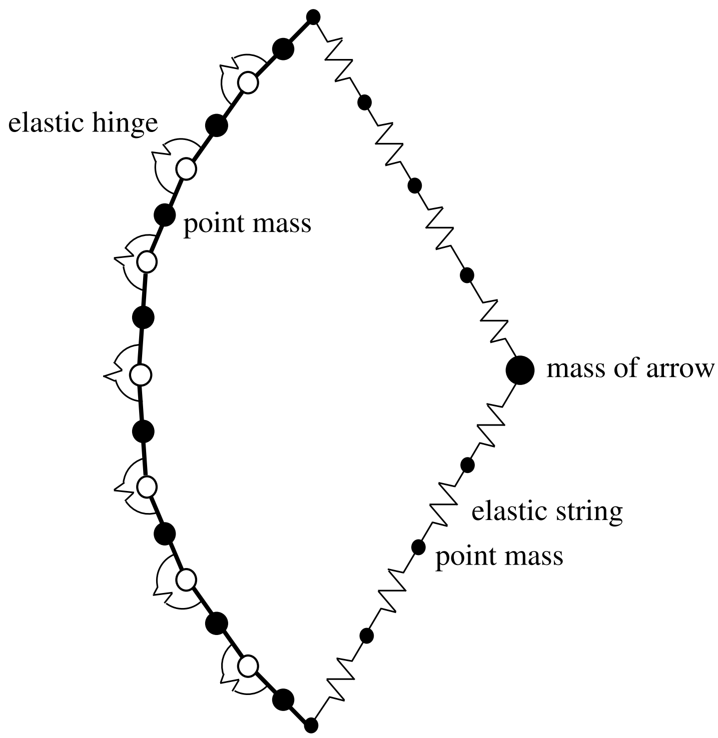


Figure 3: Our model (simplified) of a bow. Several linear elastic hinges connect rigid elements with point masses in their centre.

	elastic properties of the materials of the limbs
	specific mass of the materials of the limbs
	shape and mass of the ears, if present
	mass of the horns
	fistmele
	draw length
string:	mass of the string
	elastic properties of the string
arrow:	mass of the arrow

These design parameters determine the weight of the bow. In practice, the bowyer tillers the bow until it has the desired weight for a particular draw length. The archer on the other hand sets the fistmele or brace height by adjusting the string length.

2.4 Quality coefficients

To compare bow performance, it is necessary to introduce three quality coefficients. The static quality coefficient, q, measures how much recoverable energy is stored in the fully drawn bow; it is defined as the additional deformation energy stored in the elastic limbs and string by drawing the bow from the braced into the fully drawn position divided by the weight times the draw length. The efficiency, η, is the kinetic energy transferred to the arrow divided by the above-mentioned additional deformation energy; so it is the part of the amount of available energy that is transferred to the arrow as useful energy. The third quality coefficient, ν, is proportional to the initial velocity; the constant depends only on the weight, draw length, and mass of the limbs.

For flight shooting, the initial velocity of the arrow leaving the string is very important: the greater this velocity, the longer the maximum attainable distance. The actual distance depends also on the elevation angle (nearly 45°) and the drag of the arrow in the air. Flight arrows typically have closely cropped fletching to reduce drag. Target shooting and hunting require that the bow shoots smoothly. It is difficult to translate this feature into quantifiable terms. High efficiency is a good criterion. A heavy arrow always yields a high efficiency – and, unfortunately so, a small initial velocity and therefore a short distance. The recoil-force – the force the archer feels in the bowhand after the release – also seems important. The way this force changes in time can be calculated using the model, but it cannot be summarized by a single number.

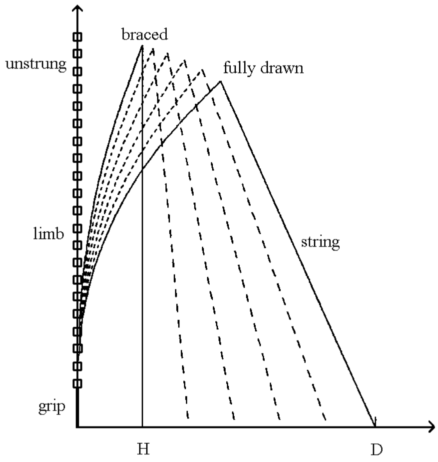
The bow should not exaggerate human error. To assess the sensitivity of a bow, its performance is calculated repeatedly with slightly different values for the design parameters. If performance is strongly dependent on a design parameter, the archer has to take care that the value of this parameter is as constant as possible. To achieve this, archers need skill as well as technique.

3 Validation of the mathematical model

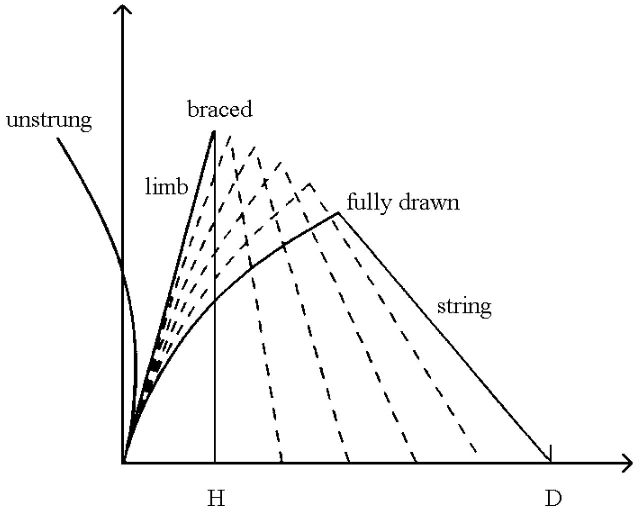
Mathematical models may be beautiful in themselves and the way to solve them interesting, but they should mimic the mechanical action of the bow and arrow well enough if they are to be used in bow design or a sensitivity study.

With calculated values, Kooi checked static action by comparing the measured weight of a replica of the longbows recovered from the *Mary Rose*, the warship of Henry VIII that sank in 1545 in The Solent, a mile outside Portsmouth, England. She was recovered in 1982 along with 139 yew longbows. Tests with these bows have demonstrated that while it is possible to string and draw the bows to 30-inches, considerable degradation within the cell structure of the wood has prevented a realistic assessment of the original weight. A replica was made by Roy King, bowyer to the *Mary Rose Trust*. Prof. P. Pratt of Imperial College, London, measured all design parameters that are required to calculate the mechanical performance of a bow. The weight of this replica was also measured. It compared very well using the predicted value calculated with the mathematical model; the differences were within 1% (Hardy 1992). These results suggest that if a good estimate of the original modulus and density can be obtained, the original mechanical performance of the longbows can be calculated from the dimensions of these recovered bows.

Data obtained with the test set-up, described extensively in Tuijn & Kooi (1992), permitted a comparison of predicted and measured arrow velocities. The dynamic action of bows could be checked in this way; a modern bow made of maple in the core and glass fibers embedded in strong synthetic resin for the back and belly was used. All the essential



a)



b)

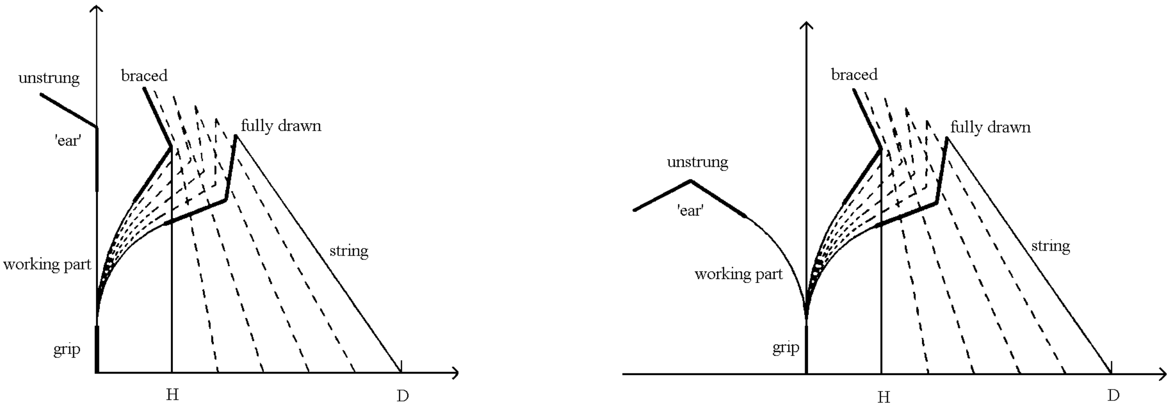
Figure 4: Non-recurve bows: Static deformation shapes (a) of the 'Klopsteg' bow and (b) of the 'Angular' bow. 'H' denotes the strung position, 'D' the full draw position of the middle of the string. The unstrung shape of the bow is denoted by the '□' symbols.

parameters listed above were measured. The density and elastic modulus of both the wood and the fiberglass and, at a number of points along the limbs, the shape of the cross-sections were measured. The results were used to determine the bending properties of the limbs. Finally, the elastic modulus and the mass of the string were measured.

As the predicted weight was too high, a knockdown factor was used for the flexural rigidity of the limbs, so that the calculated weight became equal to the measured value. The predicted amount of energy stored in the bow by drawing it from a braced position to full draw differed only slightly from the measured value. The measured efficiency was a few percent below the calculated value. In the model, internal and external damping are neglected, which explains part of this discrepancy.

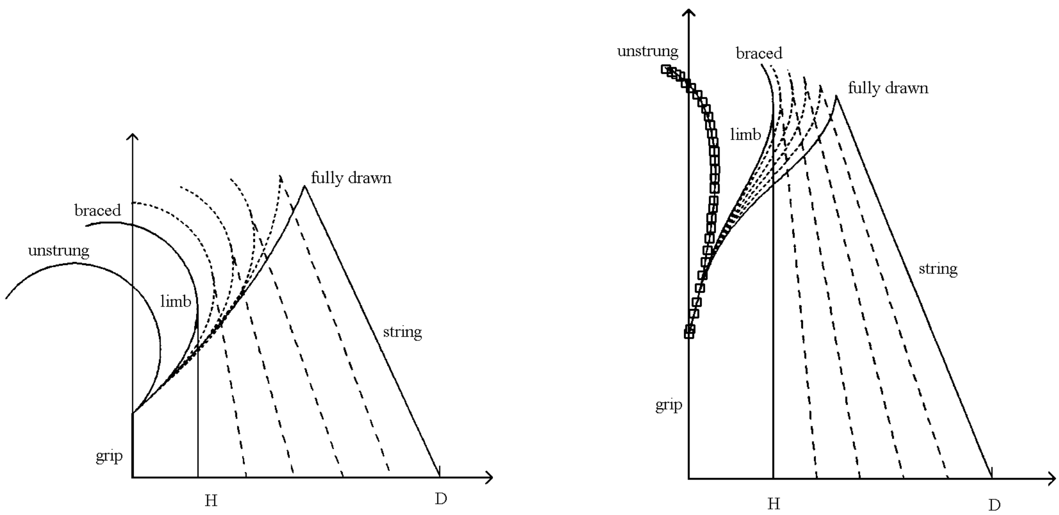
4 Classification of the bow

For this study, the classification of bows is based on the geometrical shape and the elastic properties of the limbs. In the model, the bow is assumed to be symmetric with respect to the line of aim; only one-half of the bow is considered. The bows of which the upper half are depicted in Figure 4 are called non-recurve bows; these have contact with the string only at their tips. In the static-recurve bow (Figure 5), the outermost parts of the recurved limbs (the ears) are stiff. In the braced position, the string rests on string-bridges. These string-bridges are fitted to prevent the string from slipping past the limbs when the arrow is released. When this bow is drawn, the string leaves the bridges and has contact with the limbs only at the tips. In a working-recurve bow, the limbs are also curved in the "opposite" direction in the unstrung situation (Figure 6).



a) b)

Figure 5: Static-recurve bows: Static deformation shapes (a) of the 'Persian' bow and (b) of the 'Turkish' bow.



a) b)

Figure 6: Working-recurve bows: Static deformation shapes (a) of the 'excessive-recurve' bow and (b) of the 'working-recurve' bow.

Table 1: Quality coefficients for various bows: static quality coefficient q; efficiency η and initial velocity ν. Note that the values for the working recurve 'working-recurve' bow are adapted for the masses of the arrow and string, but the stiffness of its string is about twice those of the other bows.

Bow	q	η	ν
'Klopsteg' bow	0.407	0.765	2.01
'Angular' bow	0.395	0.716	1.92
'Persian' bow	0.432	0.668	1.94
'Turkish' bow	0.491	0.619	1.99
'excessive-recurve' bow	0.810	0.417	2.08
'working-recurve' bow	0.434	0.770	2.09

The parts of a working-recurve bow near the tips are elastic and bend during the final part of the draw. When one draws this bow, the length of contact between string and limb decreases gradually until the string leaves the tip. The string remains in that position during the final part of the draw.

Representations of different types of bows form clusters in the design parameter space. To study the performance of different types of bow, the authors will start with a straight-end bow described by Klopsteg (in Hickman *et al.* 1947), the 'Klopsteg' bow; its shape for various draw lengths is shown in Figure 4a. Another non-recurve bow is the 'Angular' bow found in Egypt and Assyria; its unstrung shape (Figure 4b) implies that in the braced position the limbs and the string form the characteristic triangular shape. Two static-recurve bows, one from China or India/Persia, the 'Persian' bow, is considered along with one which resembles a Turkish flight bow described by Payne-Gallwey (1976), the 'Turkish' bow. The shapes of these bows for various draw-lengths are shown in Figure 5. One of the working-recurve bows is named for its 'excessive-recurve'; it resembles a bow described and shot by Hickman (Hickman *et al.* 1947). The other working-recurve bow is the modern one that was used for the validation of the model (Tuijn & Kooi 1992), the 'working-recurve' bow (Figure 6b).

The three quality coefficients for these types of bow are shown in Table 1. These coefficients are defined for equal weight, draw length and mass of the limbs; and the mass of the arrows and strings was the same for all reported bows. This makes an honest comparison possible. Unfortunately, the stiffness of the string of the 'working-recurve' bow is about twice that of the other bows. The static quality coefficient q for the static-recurve bows is greater than for non-recurve bows. For the efficiency η , the opposite holds: non-recurve bows are more efficient than static-recurve bows. Hence, the initial velocity ν is

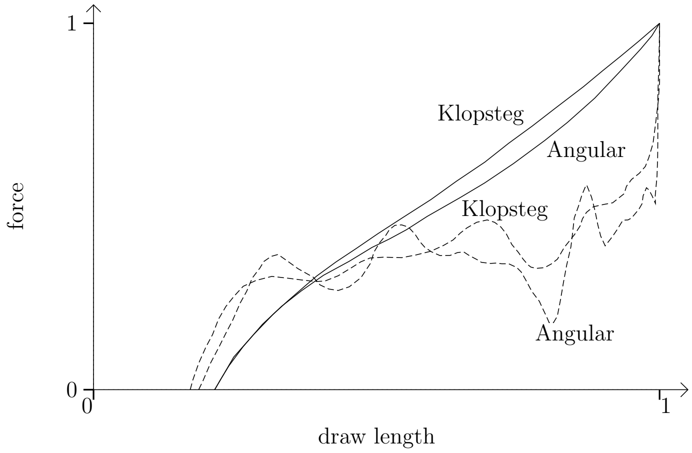


Figure 7: Static (solid line) and dynamic (dashed line) force-draw curves for the non-recurve bows: the 'Klopsteg' and the 'Angular' bow.

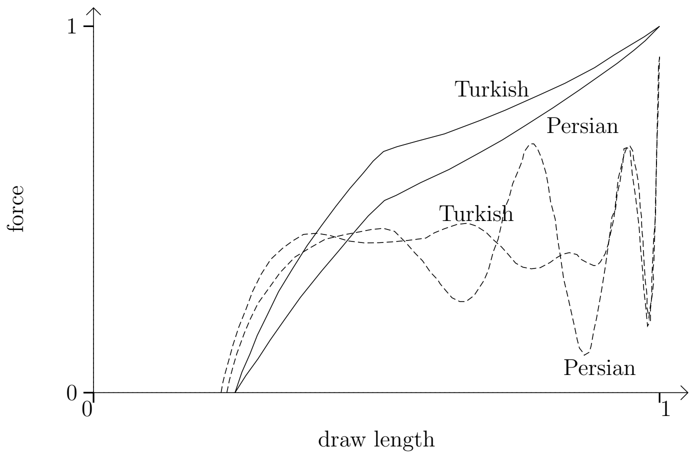


Figure 8: Static (solid line) and dynamic (dashed line) force-draw curves for static-recurve bows: the 'Persian' and the 'Turkish' bow.

about the same for these two types of bows. The bow with the excessive recurve has a very large static quality coefficient – but also low efficiency. The modern working recurve bow combines a large static quality coefficient with high efficiency; this yields the largest initial velocity.

In Figures 7-9 the calculated Static Draw Curve (SFD) and the Dynamic Force Draw (DFD) curves are shown for the non-, static- and working-recurve bows, respectively. In the SFD-curve the draw force is plotted as a function of the draw length. The area below this curve equals the amount of energy supplied by the archer in drawing from a braced position to full draw. Both force and length are scaled so that the weight and draw length equal 1, and, therefore, the area below the SFD-curve equals q.

The DFD-curve gives the acceleration force acting upon the arrow during its release as a function of the draw length. The arrow leaves the string when this force becomes zero. Due to some elasticity of the string this happens when the middle of the string has already passed its position in the braced position. The area below this curve equals the amount of kinetic energy transferred to the arrow. This area is proportional to ν^2, for the kinetic

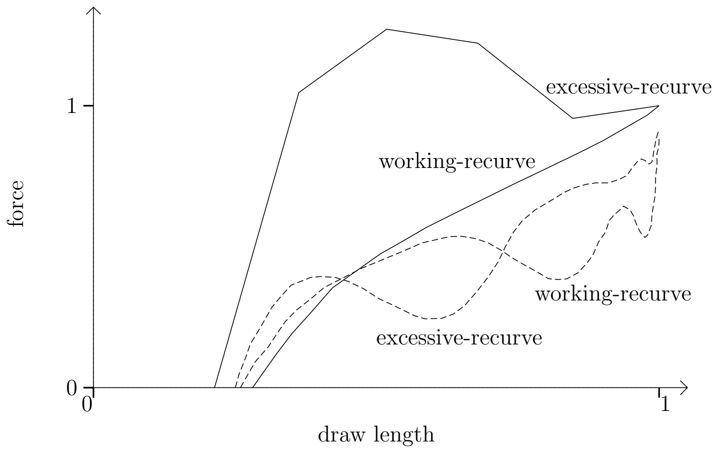


Figure 9: Static (solid line) and dynamic (dashed line) force-draw curves for the working-recurve bows: the 'excessive-recurve' and the 'working-recurve' bow.

energy of the arrow is one-half times the mass of the arrow times the velocity squared. The ratio between the areas below the DFD-curve and the SFD-curve is the efficiency η.

In discussing the ears on static-recurve bows, Hamilton (1982:71) states, "function of the 'ears', or siyahs, is well known today and no one can question the superiority of the type of bow which still holds the world record of shooting an arrow 972 yards." Later, Hamilton (1982:78) defines three ways in which the siyah contributes to improving cast by providing "leverage for the bowstring so the bow can be designed to approach maximum weight earlier in the draw allowing more energy to be stored for the cast." This statement agrees with the results of the present study. The static quality coefficient of the 'Persian' bow is larger than that of the straight-end 'Klopsteg' bow. For the static-recurve bows the SFD curves in Figure 8 show a bend at the place where the string leaves the bridges, whereas the SFD curves of both non-recurve bows do not (Figure 7). The 'Turkish' bow stores more energy in the fully drawn bow, obviously because of the recurve at the working part of the limbs. The good static performance of flight bows may result only partly from the use of the stiff ears. This is clear from the SFD curves for the working-recurve bows with no stiff ears shown in Figure 9, and especially for the 'excessive-recurve' bow with the excessive recurve.

A second point made by Hamilton (1982:78) in regard to the function of the siyah is, "upon release, the bowstring imparts its energy to the arrow more uniformly and at a higher and more sustained rate of thrust than in a ordinary bow without siyahs". This statement is not supported by the results obtained using the present model. Due to the relatively heavy ears, there is a sudden decrease in the force in the string and, by implication, in the acceleration force upon the arrow. The result of this is oscillatory behaviour (Figure 8). Consequently, the efficiency of static-recurve bows is rather low. The amplitude of the oscillations depends largely on the modulus of elasticity of the string and on the mass of the arrow relative to the mass of the ears.

Finally, Hamilton (1982:82) states, "when the bow string reaches the bridges it is in effect shortened, increasing the tension again on the bowstring and giving the arrow a final snap as it leaves the bow." The DFD curve shows that the acceleration of the arrow is rather large when the string has contact with the bridges. Notwithstanding this, the efficiency η of the 'Persian' bow, and certainly that of the 'Turkish' bow, is rather low. This implies that the initial velocity ν is not as large as one would expect on the basis of the static performance. This is caused by the relatively heavy ears. These considerations demonstrate why these bows can, inherently, be no better than long straight-end bows; a large part of the available energy remains in the vibrating limbs and string after the arrow leaves the string.

This holds true to an even larger extent for the 'excessive-recurve' bow. The string cannot slow down the lightened ends of the limbs during the final part of the acceleration of the arrow when the bow is close to its braced position (Figure 9).

The modern 'working-recurve' bow seems a good compromise between the non-recurve bow and the static-recurve bow. The recurve yields a large static quality coefficient; the flexible, and therefore light, tips of the limbs give a reasonably high efficiency.

5 Construction of the bow

What made the Turkish flight bow a superb type of bow for flight shooting? Until now the discussion has dealt with the mechanics of the bow, but not with its construction. Efficiency is affected greatly by the relative mass of the arrow when compared to that of the limbs: for a fixed mass of the arrow, the lighter the limbs the better the efficiency. This is where technology becomes important; the minimum mass of the limbs for a fixed weight and draw is determined largely by the ability of the material to store energy.

In the past, bows differed not only in shape, but also in materials. Simple bows made out of one piece of wood, straight and tapering towards the ends have been used by primitive cultures in Africa, South America and Melanesia. Bows used in prehistoric north-western Europe were also of this type (Clark 1963; Bergman 1993). In the famous English yew longbow of the 14th and 15th centuries, the different properties of sapwood and heartwood were deliberately put to use (Hardy 1992). Eskimos used wood, horn, and/or antler together with cords plaited of animal sinews and lashed to the bow's core at various points.

The Angular bows are examples of composite bows. Found in the tomb of Tut'ankhamūn (McLeod 1970), this type appears in Egyptian tomb paintings and on Assyrian monumental sculpture. Like the Angular bow, other Asian composite bows were made from wood, sinew and horn. To quote Faris & Elmer (1945:13), "Just as man is made of four component parts (bone, flesh, arteries and blood) so is the bow made of four component parts. The wood in the bow corresponds to the skeleton in man, the horn to the flesh, the sinew to the arteries, and the glue to the blood." Composite bows, used by the Mongolian peoples of eastern Asia, reached their highest development in India, Persia and Turkey. In modern bows, maple and glass or carbon fibres, embedded in strong synthetic resin are used.

Table 2 summarizes the mechanical properties for some materials used in making bows. It shows it is possible to store much more energy per unit of mass in the materials of the composite bow, sinew and horn, than in wood, the material of the simple or self bow. More-

Table 2: Mechanical properties and the energy per unit of mass referred to as $\overline{\delta}_{bv}$ for some materials used in making bows. See also Gordon (1978).

material	working stress N/m^2 10^7	elastic modulus N/m^2 10^{10}	specific mass kg/m^3	$\overline{\delta}_{bv}$ Nm/kg
steel	70.0	21.0	7800	130
sinew	7.0	.09	1100	2500
horn	9.0	.22	1200	1500
yew	12.0	1.0	600	1100
maple	10.8	1.2	700	700
glassfibre	78.5	3.9	1830	4300

over, in the composite bows these better materials were used in a more efficient manner. Sinew, very strong in tension, is used for the back. Horn, withstanding compression very well, is applied on the belly side of the limbs. A composite bow with the same mass as a simple wooden bow can have a much larger draw weight, explaining the good performance of the composite flight bow for flight shooting.

Table 3 lists the values for the weight, draw, mass and length for a number of bows described in the literature. The energy density is the calculated energy stored in the limbs per unit of mass in full draw. When materials are used to their full extent, the energy density equals the ultimate energy density $\overline{\delta}_{bv}$. In bending, fibres in the centre of the limb are not stressed at all, and only the fibres at the belly and back side are fully stressed; the energy density has to be smaller than $\overline{\delta}_{bv}$. How much smaller depends on the shape of the cross-section of the limb which generally varies along the limb. In any spring there is tensile force and all fibres are used equally. This implies that both quantities can be equal. The longbow of Table 3 is the replica *Mary Rose* bow. The values obtained for the straight-end bows look realistic. The steel bow was described by Elmer (1952:220).

Because of the stiff ears or a recurve of the working parts of the limbs, much energy is stored in the static-recurve bow. In a recurved bow, the amount of energy in the braced position is already large. This implies that the limbs must be relatively heavy in order to store this extra and unusable energy, in addition to the recoverable energy. This is the price paid for a larger static quality coefficient. As sinew and horn are relatively tough and flexible materials (Table 2), these materials fit well with the recurved shape of the unstrung bow. The values in Table 3 show that the Turkish bow is highly stressed; although very light, it is also very strong, allowing one to shoot a light arrow a long distance.

In a letter to Kooi, E. McEwen states: “Pope did not properly test his larger ‘Tartar’ (actually Manchu-Chinese) bow. He only drew it 36-inches and bows of this type and size

Table 3: Parameters for a number of bows and an estimation of $\overline{d}_{bv}$ the weight times draw length divided by the mass of one limb. For comparison the estimated values for $\overline{\delta}_{bv}$ are also given. When materials are used to their full extend, the energy density equals the ultimate energy density $\overline{\delta}_{bv}$.

Type	weight	draw	mass	length	energy density	$\overline{\delta}_{bv}$	Reference
	N	m	kg	m	Nm/kg	Nm/kg	
flatbow	155	0.7112	0.325	1.829	180	1100	(Hickman *et al.* 1947)
longbow	465	0.746	0.794	1.874	230	1100	(Hardy 1992)
steelbow	170	0.7112	0.709	1.689	90	130	(Elmer 1952)
Tartar	460	0.7366	1.47	1.88	135	2000	(Pope 1974)
Turkish	690	0.7112	0.35	1.14	1540	2000	(Payne-Gallwey 1976)
modern	126	0.7112	0.29	1.703	220	3000	(Tuijn & Kooi 1992)

are made to draw as much as 40-inches". Pope only mentions the weight for a draw length of 29-inches. As the energy density is very low, the materials of this bow are only partly used as McEwen suggests.

In the modern bow, there is a surplus of material near the riser section. This affects efficiency only slightly, as this part of the limb hardly moves and therefore the kinetic energy involved is small. In this bow, a rather large amount of unrecoverable energy in the braced position puts a constraint on the amount of recurve; it is perhaps more important that the efficiency of working-recurve bows decreases with increasing recurve. The mechanical properties of the materials of these bows, however, are much better than those of the ancient composite bows; the modern bow now holds the longest flight shooting record.

6 Discussion and conclusions

The mathematical modelling approach proved its potential in the *Mary Rose* project; the original weight could be estimated well from the measured parameters.

Knowledge gained from this project makes possible a better assessment of the weight of the longbows used during the Hundred Years War. Based on the mechanical interaction of the arrow and the bow, calculations suggested that the heavy 60-gram war arrow used at Agincourt in 1415 could have been shot from bows with draw weights of over 450 N (Rees 1993). This seemed an unreasonably large value; nowadays, only few archers can draw bows of such a great weight. Present-day experience implied a figure closer to 350 N. The high value, over 450 N, was confirmed by the study of the longbows recovered from the *Mary Rose* using the present mathematical model. Hardy (1992:218) states, "young, fit men in constant practice chosen for well-paid military service from a nation to whom the shooting of longbows had been second nature, could use the *Mary Rose* bows".

The results presented above indicate that the initial velocity is about the same for all types of bow under similar conditions, that is, the same draw weight, draw length and mass of the limbs, while the mass of the arrow and string and the stiffness of the string are also the same. So, within certain limits, the design parameters that determine the mechanical action of the bow–arrow combination appear to be less important than is often claimed. The authors endorse a view one could call holistic. It is not always possible to isolate a single feature that accounts solely for a good or bad performance of the whole bow, as Hamilton (1982) believed. According to Rausing (1967), who studied the development of the composite bow, the fact that the static quality coefficient of the short static-recurve bow is larger than that of the short straight bow, disposes of the following statement: "the composite bow has no inherent superiority over the wooden self-bow, so long as the latter was made from the most favorable kinds of timber and expertly used". The data obtained with our mathematical model suggest – if the word inherent has the same meaning as above – this statement is true. A combination of many technical factors made the composite flight bow better for flight shooting, not only a larger static quality coefficient.

The results of this research further indicate that the development of archery equipment may not be a process involving progressive improvements in performance. Rather, each design type represents one solution to the problem of creating a mobile weapon system capable of hurling lightweight projectiles. While a composite bow displays considerable design and technological innovations when compared to a self bow, it will not necessarily shoot an arrow farther or faster. Performance criteria such as those applied by Pope and Hamilton ignore the fact that a good or bad bow may only be gauged within the context of the functional requirements of the archer.

References

[1] C. A. Bergman. The development of the bow in western Europe: A technological and functional perspective. In G. L. Peterkin, H. M. Bricker, and P. Mellars, editors, *Hunting and Animal Exploration in the Later Palaeolithic and Mesolithic of Eurasia*, volume 4, chapter 7, pages 95–105. Archeological Papers of the American Anthropological Association, 1993.

[2] J. G. D. Clark. Neolithic bows from Sommerset, England and the prehistory of archery in north-western Europe. *Proceedings of the Prehistoric Society*, 29:50–98, 1963.

[3] R. P. Elmer. *Target Archery.* Hutchinson's Library of sports and pastimes, London, 1952.

[4] N. A. Faris and R. P. Elmer. *Arab Archery.* Princeton University Press, Princeton (NJ), 1945.

[5] J. E. Gordon. *Structures or Why Things Don't Fall Down.* Pelican, Harmondsworth, 1978.

[6] T. M. Hamilton. *Native American Bows.* Missouri Archaeological Society, Columbia (Ms), 1982.

[7] R. Hardy. *Longbow.* Patrick Stephens, Sparkford, Nr Yeovil, Somerset, 1992.

[8] C. N. Hickman, F. Nagler, and P. E. Klopsteg. *Archery: the technical side.* National Field Archery Association, Redlands (Ca), 1947.

[9] P.E. Klopsteg. *Turkish archery and the composite bow.* Simon Archery Foundation, Manchester, 1987.

[10] B. W. Kooi. On the mechanics of the bow and arrow. *Journal of Engineering Mathematics*, 15:119–145, 1981.

[11] B. W. Kooi. *On the Mechanics of the Bow and Arrow.* PhD thesis, Rijksuniversiteit Groningen, 1983.

[12] B. W. Kooi. On the mechanics of the modern working-recurve bow. *Computational Mechanics*, 8:291–304, 1991.

[13] B. W. Kooi. The design of the bow. *Proc. Kon. Ned. Akad. v. Wetensch.*, 97(3):1–27, 1994.

[14] B. W. Kooi and J. A. Sparenberg. On the static deformation of a bow. *Journal of Engineering Mathematics*, 14:27–45, 1980.

[15] E. McEwen, R. L. Miller, and C. A. Bergman. Early bow design and construction. *Scientific American*, 264(6):50–56, June 1991.

[16] W. McLeod. *Composite bows from the Tomb of Tutankhamūn.* Oxford University Press, Oxford, 1970.

[17] R. L. Miller, E. McEwen, and C. A. Bergman. Experimental approaches to ancient Near Eastern archery. *World Archaeology*, 18:178–195, 1986.

[18] Sir R. Payne-Gallwey. *The Crossbow.* Holland Press, London, 1976.

[19] S. T. Pope. *Bows and Arrows.* University of California Press, Berkeley (CA), 1974.

[20] G. Rausing. *The Bow.* Acta Archaeologica Lundensia. CWK Gleerups Förlag, Lund, Sweden, 1967.

[21] G. Rees. The longbow's deadly secrets. *New Scientist*, pages 24–25, June 5 1993.

[22] C. Tuijn and B. W. Kooi. The measurement of arrow velocities in the students' laboratory. *European Journal of Physics*, 13:127–134, 1992.