Bow–arrow interaction in archery [1]

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Abstract

A mathematical model of the flight of the arrow during its discharge from a bow has been proposed by Pękalski (1990). His description of the model was incomplete; in this paper we will give a full description of the model. Furthermore we propose some improvements which make his model more consistent with reality. One achievement is the modeling of contact of the arrow and grip; the pressure button is modeled as a unilateral elastic support. The acceleration force acting upon the arrow during the launch is predicted by an advanced mathematical model of bow dynamics. There is a satisfactory conformity of the simulation data with the experimental data. The new model predicts that the arrow leaves the pressure button before it leaves the string, as reported in the literature. The ability to model arrow dynamics gives understanding and can be used to improve the adjustment of the bow–arrow system for optimal performance.

Keywords: Archery, arrow motion, Archer's Paradox, shooting, simulation.

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Introduction

Pękalski (1990) introduced methods and research techniques in archery. He discussed the following subjects:

- A mathematical model for an arrow's movement during the interaction with a bow. The governing equation is a linear fourth order parabolic partial differential equation with boundary and initial conditions.
- A mechanical model of an archer—bow—arrow system; this is a mechanical shooting machine which gives reproducibility.
- Pękalski filmed (1000-2500 Hz high-speed 16-mm cine film) the arrow release by a female member of the Polish National team and an arrow released from the shooting machine. Fig. 1 shows the arrow's transverse motions taken from the film made with the camera viewing the archer from above.

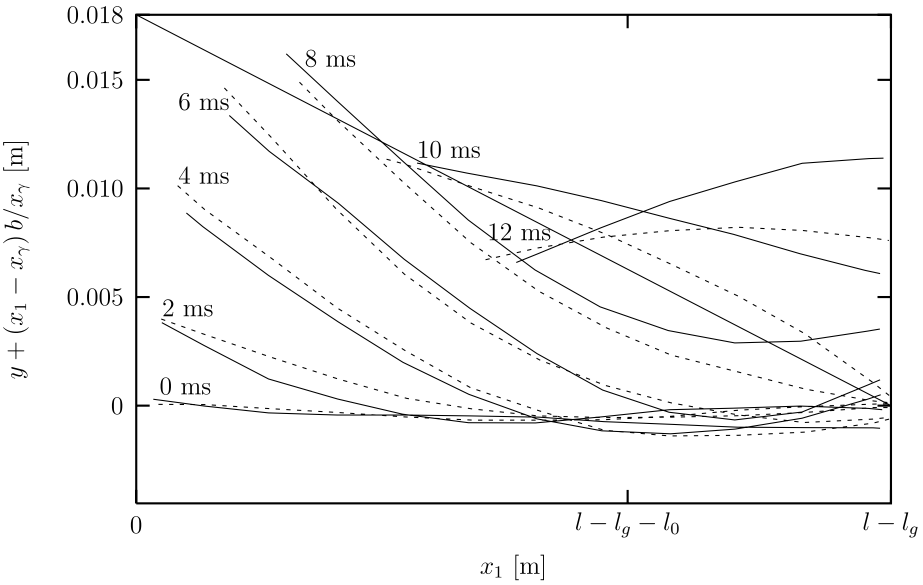

Figure 1: Curves of arrow's deflection on the basis of experimental data, line (- - -) and on the basis of Pękalski's mathematical model (——), $y + (x_1 - x_\gamma)\,b/x_\gamma$ every 2 ms after release, after (Pękalski, 1990). The shape of the arrow is shown in a coordinate system fixed to the bow; the arrow in fully draw situation is on the horizontal axis and the place of the grip of the bow is in the point $(l-l_g, 0)$. The solid line between points $(0, b)$ and $(l-l_g, 0)$ indicates the median plane of the bow.

We comment on the mathematical model for an arrow's movement during the interaction with a bow described by Pękalski (1987;1990). Pękalski made simplifying assumptions in order to be able to solve the governing equations (a linear partial differential equation) using a Fourier's method with Krilov's function base. To give insight into these assumptions

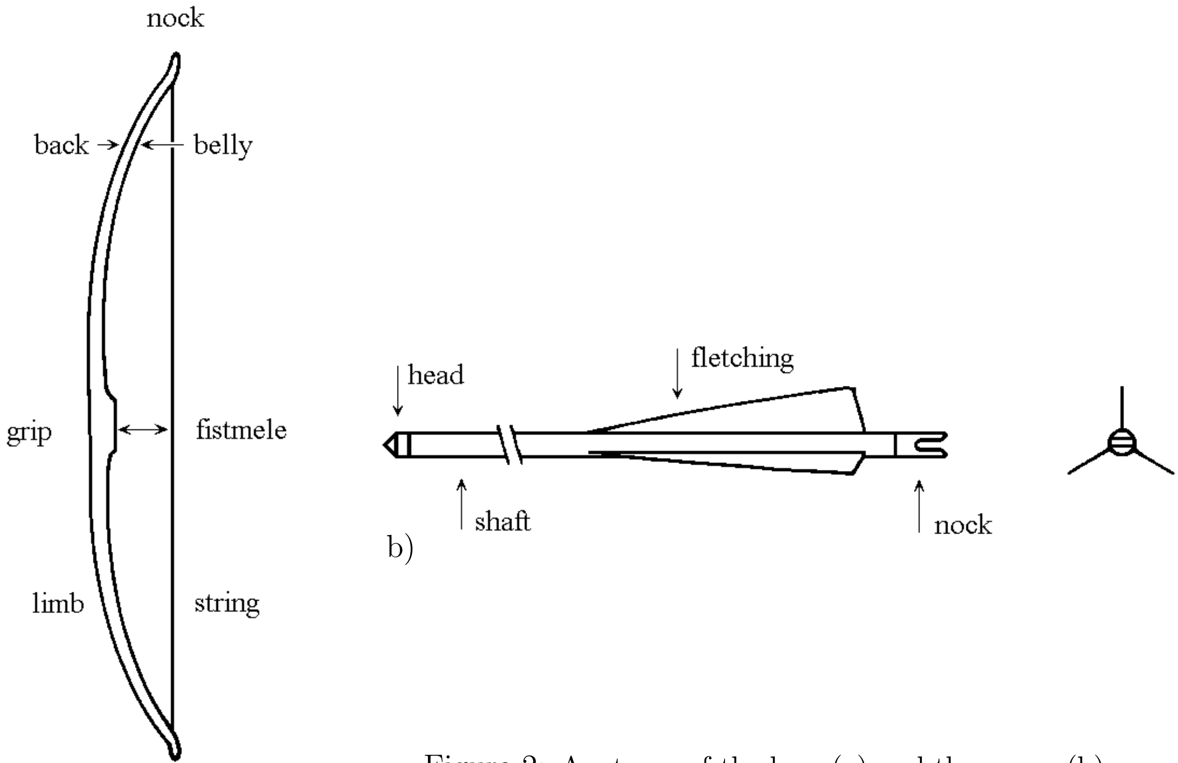

Figure 2: Anatomy of the bow (a) and the arrow (b).

we discuss his theory in its entirety. In fact his analysis was the incentive for reconsidering the problem. We present a more accurate model. In this article we focus on biomechanical aspects and the use of the knowledge gained in the sport of archery, while the mathematical aspect are presented elsewhere, Kooi and Sparenberg (1997). The system obtained is nonlinear; it has to be solved numerically. A finite difference technique has been used to solve the nonlinear partial differential equations with initial and (moving) boundary conditions. The model predictions are compared with data from literature.

All modern bows have an arrow rest on which the arrow is supported vertically and the arrow is horizontally unilaterally supported by a pressure button (shock absorber) with a built-in spring (Bolnick *et al.*, 1993, page 41). Gallozzi *et al.* (1987) and Leonardi *et al.* (1987) showed experimentally that the arrow is in contact with the grip for some period shorter than the duration of the contact between the arrow and the string. Hence, after some moment the arrow is free from the grip while it is still being accelerated by the string until it separates from the string. This phenomenon is also clearly seen in the recent videos (Sanchez, 1989) with Olympic gold medalist Jay Barrs filmed and (Rabska and van Otteren, 1991). This experimental observation is also predicted by the new model.

The simulation results give understanding of the arrow dynamics. When detailed experimental data are extracted from video analysis the model can be used to improve the adjustment of the bow–arrow system for optimal performance.

The bow and arrow

In essence the bow proper consists of two elastic limbs, often separated by a rigid middle part, the grip. The bow is braced by fastening a rather stiff string between both ends of the limbs (Fig. 2). In this figure also the anatomy of the arrow is shown. The arrow

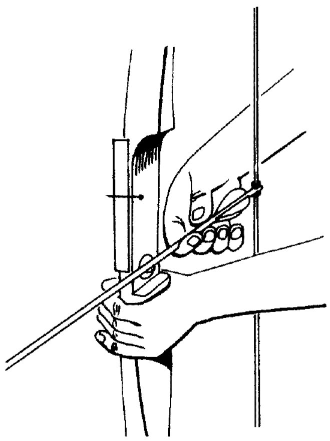

Figure 3: The arrow is set on the string, after (Baier *et al.*, 1976, page 36). The nock is provided with a groove in which the string slightly sticks when the arrow is set on the string. The arrow is supported vertically by the arrow rest fixed to the bow grip and horizontally unilaterally by a pressure button with a built-in spring which is not shown in the figure.

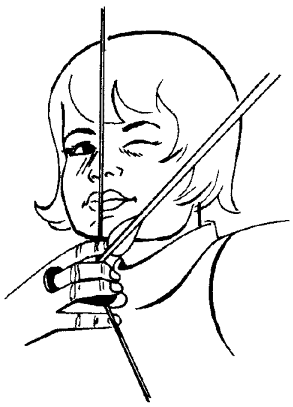

Figure 4: With the Mediterranean release the first three fingers draw the string, while the engaged arrow rests between the first and the second fingers. The string is located over the second interphalangeal joints. In full draw the second joint of the index finger of the drawing hand touches under the center of the chin, after (Baier *et al.*, 1976, page 37).

consist of a shaft, arrow head at the front-end, fletching and nock at the rear-end. The nock is provided with a groove in which the string slightly sticks when the arrow is set on the string (nock) (Fig. 3). Then the archer extends the bowarm (extend) and pulls the bow from braced situation into full draw (draw). We assume the so called Mediterranean release. The first three fingers draw the string, while the engaged arrow rests between the first and the second fingers (Fig. 4). The string is located over the second interphalangeal joints. In full draw the index finger of the drawing hand touches under the center of the chin (anchor). This completes the static action in which potential energy is stored in the elastic parts of the bow.

After aiming (hold, aim), the arrow is loosed by extending the pull fingers (release). The force in the string accelerates the arrow and transfers part of the available energy as kinetic energy to the arrow. Meanwhile the bow is held in its place and the archer feels a recoil force in the bow hand (afterhold). After the arrow has left the string the bow returns to the braced position because of damping. Klopsteg (1992) deals with the physics of bows and arrows and Kooi (1991) proposed a mathematical model of the bow. The reader is referred to (Axford, 1995) for a detailed description of the interrelationship between the anatomy of the human body and the anatomy of the bow and arrow.

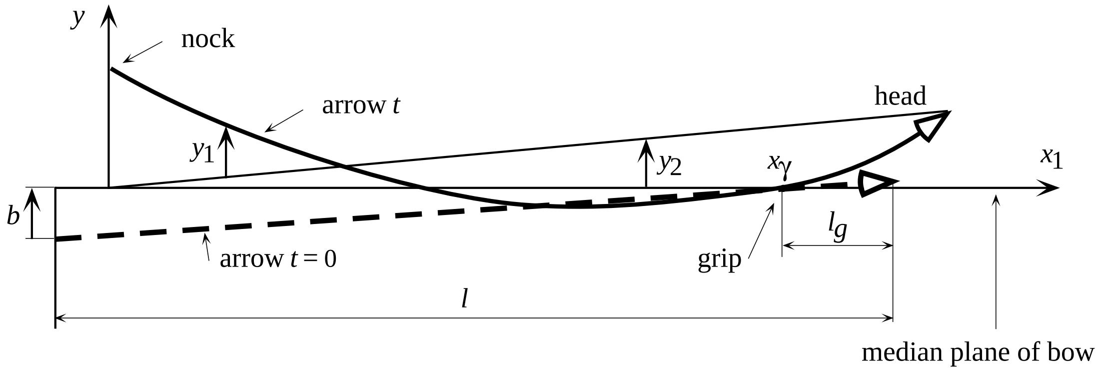

Figure 5: Solid line shows the shape of the arrow, the dashed line is the shape at $t = 0$, which is out of the median plane of the bow. The displacement out of the median plane is $y = y_1 + y_2$.

The following sequence of shooting motions are distinguished in (Baier *et al.*, 1976; Bolnick *et al.*, 1993): stand, nock, extend, draw, anchor, hold, aim, release and afterhold. In (Leroyer *et al.*, 1993) three phases are mentioned: stance (stand), arming (nock, extend, draw, anchor) and the sighting (hold, aim). They analyzed the displacement pull-hand measurement during this final push-pull phase of the shoot. Pękalski (1990) divided the ballistics of the arrow in two phases, *phase 1* "internal ballistics": the interaction between the arrow and the archer-bow system until the arrow leaves the string, and *phase 2* "external ballistics": which lasts from the end of phase 1 until the arrow hits the target. This paper deals with *phase 1* extended to the moment the arrow passes the grip to be able to study the so-called 'Archer's Paradox', (Klopsteg, 1992; Baier *et al.*, 1976); the arrow "oscillates its way" past the bow without a slap of the rear-end against the grip of the bow.

With respect to the dynamics of the arrow the following dimensions of the arrow are important. We use the notation introduced by Pękalski (1990). The total mass of the arrow, m_s, is the sum of the mass of the arrow shaft, m_k, arrow head, m_g, fletching, m_p, and nock, m_n. The length of the arrow is denoted by l. It is measured from the rear-end, the nock, to the fore-end, the head (Fig. 5). The position of the arrow with respect to the bow is determined by the distance, l_g, between the arrow head and the grip of the bow in full draw. The distance between the arrow nock and the grip is denoted by l_n. In the braced position this distance is denoted by l_0. The dimensions of a modern bow–arrow equipment are given in Table 1.

Pękalski's model

Pękalski modeled the movements of the arrow during the launch of the arrow. While being accelerated, the arrow vibrates in the horizontal plane.

Forward movement

The draw-force F_n is assumed to be proportional to the draw-length l_n minus the brace height l_0

$$F_n = k_{xs}(l_n - l_0) \ . \tag{1}$$

The acceleration force is assumed to be

$$E_n = \eta_x k_{xs}(l_n - l_0) \ . \tag{2}$$

That is, the bow is modeled as a simple linear spring with stiffness k_{xs} and efficiency η_x. The movement of the arrow towards the target during the release from the bow, follows from this force when the arrow is approximated by a particle with mass m_s. Consequently the acceleration of the arrow equals E_n/m_s.

Transverse movement in the horizontal plane

The transverse movement of the arrow in the horizontal plane (vertical movements are neglected) is composed of two components

$$y(x_1, t_1) = y_1(x_1, t_1) + y_2(x_1, t_1) \ , \tag{3}$$

where x_1 is the length coordinate along the arrow measured from the rear-end, $y_1(x_1, t_1)$ is a vibrational movement and $y_2(x_1, t_1)$ is a rotational motion of the arrow around the nock (Fig. 5). The second independent variable t_1 denotes time. So, the arrow is placed in a Cartesian coordinate system (x_1, y), the origin coinciding moving along the median plane of the bow.

The vibrating movement satisfies the well-known *beam equation*

$$EJ \frac{\partial^4 y_1}{\partial x_1^4}(x_1, t_1) + A\rho \frac{\partial^2 y_1}{\partial t^2}(x_1, t_1) = 0 \ . \tag{4}$$

The so-called Euler-Bernoulli equation is assumed. Then the curvature $\partial^2 y_1/\partial x_1^2$ is proportional to the bending moment, the proportionality constant being the flexural rigidity (bending stiffness), denoted by EJ, where E is Young's modulus of the material and J is the second area moment of inertia of the cross-section with respect to the neutral axis of the arrow. The shear force equals $-EJ\, \partial^3 y_1/\partial x_1^3$. Then eqn (4) follows from the equations of motion of a typical element of the beam, where ρ is the density and A the cross-sectional area (Timoshenko *et al.*, 1974). For a modern tubular arrow shaft the area A and the second moment of inertia J are given by

$$A = \pi\big(d^2 - (d - 2g)^2\big)/4 \ \text{ and } \ J = \pi\big(d^4 - (d - 2g)^4\big)/64 \ . \tag{5}$$

where d and g are the external diameter and the shaft wall thickness, respectively.

The boundary conditions at the nock, $x_1 = 0$, are

$$\frac{\partial^2 y_1}{\partial x_1^2}(0, t_1) = 0 \ , \ EJ\frac{\partial^3 y_1}{\partial x_1^3}(0, t_1) + \eta_y k_{ys}\, y_1(0, t_1) = 0 \ , \tag{6}$$

where k_{ys} is the static transverse elasticity of the bow and η_y is the associated efficiency. The first equation means that the bending moment is zero at the nock. The second states that the shear force equals the force in the spring which represents the transversal elasticity of the bow.

The boundary conditions at the arrowhead, $x_1 = l$, are

$$\frac{\partial^2 y_1}{\partial x_1^2}(l, t_1) = 0 \ , \ y_1(l, t_1) = 0 \ . \tag{7}$$

The first equation means that the moment at the arrow head is zero and the second equation means that the transversal displacement is zero. That is, the arrowhead is placed in a hinge-like joint.

The initial conditions for $y_1(x_1, t_1)$ at $t_1 = 0$ are

$$y_1(x_1, 0) = \frac{b}{l}(x_1 - l) \ , \ \frac{\partial y_1}{\partial t_1}(x_1, 0) = 0 \ . \tag{8}$$

The nock is a distance b out of the median plane of the bow, for a right-handed archer to the right. The deflection b of the nock is a parameter which depends on the archer's technique.

The second movement $y_2(x_1, t_1)$, is the rotation of the arrow such that $y_2(0, t_1) = 0$. Let t_f denote the instant the arrow becomes free from the grip, then $y_2(x_1, t_1)$ is determined during $0 \leq t_1 \leq t_f$ by the requirement that the arrow remains in contact with the grip where the x_1 coordinate is denoted by $x_\gamma(t_1)$. Thus

$$y(x_1, t_1) = y_1(x_1, t_1) - \frac{x_1}{x_\gamma(t_1)} y_1(x_\gamma(t_1), t_1) \ , \tag{9}$$

where $y(x_\gamma(t_1), t_1) = 0$ since the arrow is in contact with the grip. The function $x_\gamma(t_1)$ follows from the forward movement of the arrow

$$\ddot{x}_\gamma(t_1) = -\, E_n(t_1)/m_s \ , \quad \dot{x}_\gamma(0) = 0 \ , \quad x_\gamma(0) = l - l_g \ , \tag{10}$$

The acceleration of the point of contact on the grip equals the acceleration of the arrow.

In Pękalski's theory t_f is determined by the moment at which the transverse velocity of the arrowhead $V_y(t_1) = \partial y/\partial t_1(l, t_1)$ is maximum. For $t_1 \geq t_f$ Pękalski assumes that the velocity V_{ymax}, remains unaltered. Then, the following total movement $y(x_1, t_1)$ occurs after the arrow looses contact with the grip

$$y(x_1, t_1) = y_1(x_1, t_1) + \frac{x_1}{l}\big(y(l, t_f) + (t_1 - t_f)V_{ymax}\big) \ . \tag{11}$$

Pękalski solved the linear partial differential equation for the variable $y_1(x_1, t_1)$, eqn (4), with the boundary conditions (6) and (7), together with the initial condition (8), by means of the Krilov function technique. Substitution of this solution in eqn (9) for $0 \leq t_1 \leq t_f$, and eqn (11) for $t_1 \geq t_f$, yields the shape of the arrow.

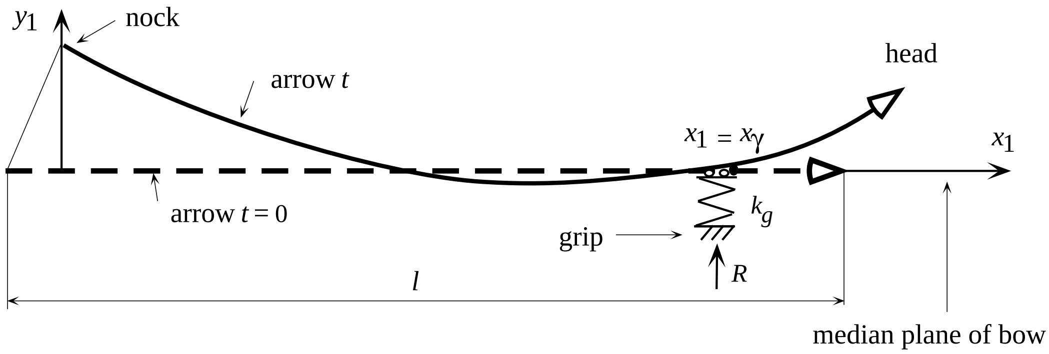

Figure 6: Solid line shows the shape of the arrow when it clears the finger tab. The dashed line shows the shape at $t = 0$, which is in the median plane of the bow.

Kooi/Sparenberg model

In this section the model proposed in Kooi and Sparenberg (1997) is described. The deviations from Pękalski's model are emphasized and the motivations for the improvements are given.

Forward movement

The simple linear model for the bow Pękalski used, is replaced by the model described in Kooi (1991). Fig. 7 gives the calculated Static Force Draw curve (SFD), denoted by F, of a modern competition bow. It is the force needed to draw an arrow to a specified draw length. This curve is approximated by that of the linear spring, F_n, which is proportional to the actual draw length and so that the areas below both curves (representing the available energy to be transferred to the arrow) are equal. Pękalski assumed the Dynamic Force Draw curve (DFD) (the nonlinear acceleration force, E, acting upon the arrow) to be the efficiency η_x times the values of the SFD curve or $E_n = \eta_x F_n$. In Fig. 7 we show also the calculated DFD curve, for a modern bow. The mathematical model and the applied numerical technique are described elsewhere (Kooi, 1991).

Transverse movement in the horizontal plane

In this new model the displacement $y_1(x_1, t_1)$ of the arrow is again measured from the median plane of the bow, see Fig. 6. The longitudinal force, $H(x_1, t_1)$ (positive for tensile forces), due to the acceleration force is taken into account

$$EJ\frac{\partial^4 y_1}{\partial x_1^4} - \frac{\partial H \partial y_1/\partial x_1}{\partial x_1} + A\rho\frac{\partial^2 y_1}{\partial t_1^2} = 0 \ . \tag{12}$$

The longitudinal force is given by

$$H(x_1,t_1) = -\frac{\rho C(l-x_1)+m_g}{m_s}\,E_n(t_1)\ . \qquad (13)$$

One of the main differences between Pękalski's model and the model proposed now is the way the release is modeled. With the Mediterranean release the first three fingers draw the string, located over the second interphalangeal joints. In the presented model, starting in the median plane, the string slips of the finger tips making the nock of the arrow off-center to the median plane of the bow, see also (Axford, 1995; Baier *et al.*, 1976, page 75). In mathematical terms this means that the path of the nock $y_1(0,t_1)$ is prescribed for the period the finger tips contact the nock. We have taken for the length of the contact line 0.0035 m in the longitudinal x_1-direction and 0.00229 m in the transverse y_1-direction. These values were obtained by trial and error in order to get good correlation with experimental data. These parameters depend on the archer's technique. Fig. 6 shows that the arrow is initially in the median plane, so the initial conditions are

$$y_1(x_1,0) = 0\ \ ,\ \ \frac{\partial y_1}{\partial t_1}(x_1,0) = 0\ , \qquad (14)$$

that is the transverse velocity is zero.

It follows from eqns (8) and (6) that directly after release the nock in Pękalski's model has a velocity toward the median plane by the transverse elasticity of the bow because in his model the nock of the arrow starts off-center ($b \leq 0$). The same happens with the nock in this model when the string slips of the finger tips. Hence, just after release there is a resemblance of the motions of the nock of the arrow in both models. In this model the artificial initial position of the arrow off-center, b, equivalent to a bow's torsion around the vertical axis, is not needed.

After the arrow has left the finger tips the boundary conditions at the place where the nock of the arrow $x_1 = 0$ resemble those in Pękalski's model eqns (6)

$$\frac{\partial^2 y_1}{\partial x_1^2}(0,t_1) = 0\ \ ,\ \ EJ\frac{\partial^3 y_1}{\partial x_1^3}(0,t_1) - H\frac{\partial y_1}{\partial x_1}(0,t_1) + \eta_y k_{ys}\, y_1(0,t_1) = 0\ , \qquad (15)$$

where the mass of the nock m_n is neglected. After the arrow has left the string, the instant denoted by t_l, the last term in the second equations disappears, that is the transverse force becomes zero.

Another difference is the modeling of the contact between the arrow and the grip. As in Pękalski (1990) the protrusion of the arrow's rest, denoted by y_p, is zero. In this model the contact force is taken into account explicitly, making the third order spatial derivative of the deflections discontinuous at the place of contact, $x_1 = x_\gamma(t_1)$, which is a function of time due to the forward motion of the arrow eqn. (10) where $E_n(t_1)$ is replaced by $E(t_1)$, see Fig. 7.

The contact force between arrow and grip, $R(t_1)$, is proportional to the discontinuity of the third order partial derivative

$$R(t_1) = \lim_{x_1\downarrow x_\gamma(t_1)} \frac{\partial^3 y_1}{\partial x_1^3}(x_1,t_1) - \lim_{x_1\uparrow x_\gamma(t_1)} \frac{\partial^3 y_1}{\partial x_1^3}(x_1,t_1)\ . \qquad (16)$$

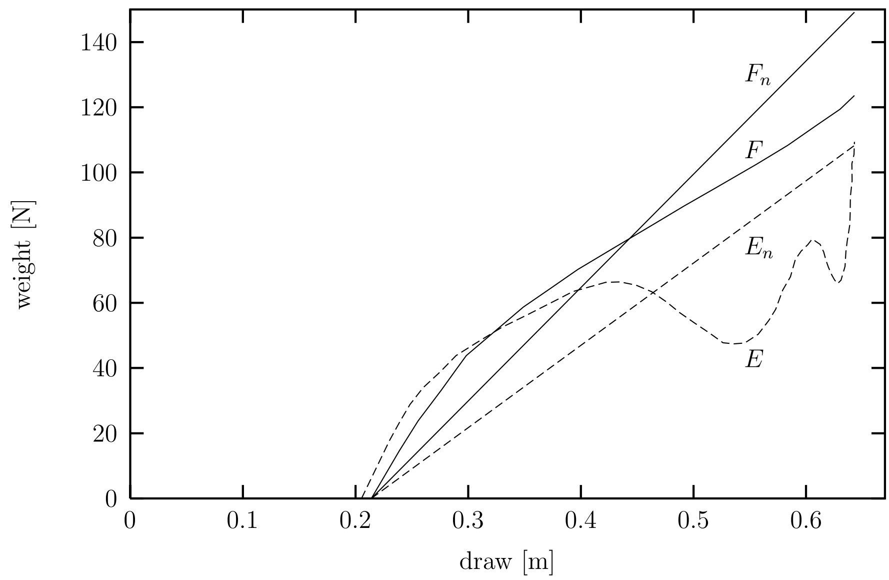

Figure 7: Calculated Static Force Draw curve, F and Dynamic Force Draw curve, E for a modern working-recurve bow. The SFD curve F_n and the DFD curve, E_n of the associated linear-spring bow used by Pękalski (1990).

This force acting in the transverse y_1-direction is also equal to the force in the spring of the pressure button

$$R(t_1) = \begin{cases} -k_g y_1(x_\gamma(t_1), t_1) & \text{if} \quad y_1(x_\gamma, t_1) \geq 0 \\ 0 & \text{if} \quad y_1(x_\gamma, t_1) > 0 \quad , \quad t_1 \geq t_f \ . \end{cases} \tag{17}$$

where k_g is the spring constant of the pressure button. The moment the arrow has contact with the grip for the last time is denoted as t_f.

The boundary conditions at the tip of the arrow, $x_1 = l$, read

$$\frac{\partial^2 y_1}{\partial x_1^2}(l, t_1) = 0 \ , \ EJ\frac{\partial^3 y_1}{\partial x_1^3}(l, t_1) - H\frac{\partial y_1}{\partial x_1}(l, t_1) - m_g\frac{\partial^2 y_1}{\partial t^2}(l, t_1) = 0 \ . \tag{18}$$

The mass of the arrow head m_g is taken into account as a point mass.

This completes description of the mathematical model. In contrast with Pękalski's model, Fourier's method can not be used to analyze this system of nonlinear (due to the contact problem) partial differential equations with boundary and initial conditions. In Kooi and Sparenberg (1997) a finite difference technique (Mitchell and Griffiths, 1980) was proposed to solve the equations numerically; the presented results were calculated with that method.

Results

In Fig. 1 we give the shapes calculated by Pękalski for the standard arrow shot with the standard bow for every 2 milliseconds (ms) after release. The parameter setting for the

Table 1: Values for the parameters of the Easton 1714X7 (Aluminum 7178) arrow after Pękalski (1987). The bow is a Hoyt pro medalist T/D, 66 inch, 34 lbs. For the standard arrow-bow combination $\eta_x = \eta_y = 0.75$.

parameter	description	unit	value
arrow			
l	length	m	$6.7\,10^{-1}$
d	external diameter	m	$6.75\,10^{-3}$
g	wall thickness	m	$3.56\,10^{-4}$
ρA	mass per unit length	$\mathrm{kg\,m^{-1}}$	$2.0\,10^{-2}$
EJ	flexural rigidity	$\mathrm{N\,m^2}$	$2.088\,10^{2}$
m_s	mass of arrow	kg	$1.85\,10^{-2}$
m_k	mass of shaft	kg	$1.35\,10^{-2}$
m_g	mass of head	kg	$4.0\,10^{-3}$
m_p	mass of fletching	kg	$1.0\,10^{-3}$
m_n	mass of nock	kg	$4.0\,10^{-4}$
bow			
k_{xs}	bow stiffness	$\mathrm{N\,m^{-1}}$	$3.42\,10^{2}$
k_{ys}	transverse stiffness	$\mathrm{N\,m^{-1}}$	$2.7\,10^{2}$
l_0	initial drawing length	m	$2.25\,10^{-1}$
l_n	drawing length	m	$6.45\,10^{-1}$
$F_n(l_n)$	drawing force	N	$1.43\,10^{2}$
arrow & bow			
k_g	spring constant of pressure button	$\mathrm{N\,m^{-1}}$	$6.22\,10^{2}$
l_g	distance between head and grip	m	$2.5\,10^{-2}$

standard arrow and bow are given in Table 1. The solid line between the points $(0, b)$ and $(l - l_g, 0)$ indicates the median plane of the bow. Pękalski used a nonlinear least-square regression technique to estimate the following parameter values: $\eta_x = 0.76$, $\eta_y = 0.71$ and $b = -0.018$ m.

Fig. 8 gives the shapes $y_1(x_1, t_1)$ calculated with the new model for the standard arrow shot with the standard bow for every 2 milliseconds (ms) after release until the nock passes the grip. Fig. 9 gives the shapes of the arrow predicted by our model but now with the acceleration force E of a modern bow instead of the linear one E_n. The modern bow was also used in the experiments described by Tuijn and Kooi (1992); the measured efficiency was only a few percent below the value predicted by the model. For both models of the bow, in Fig. 10 the contact force between the arrow and the grip, denoted by $R(t_1)$, are shown. The model predicts that shortly after release the arrow leaves the button again for a very short period, that is the recoil force is zero in that period. The contact ends at $t_f < t_l$, so before the arrow leaves the string. The displacement of the arrow nock for

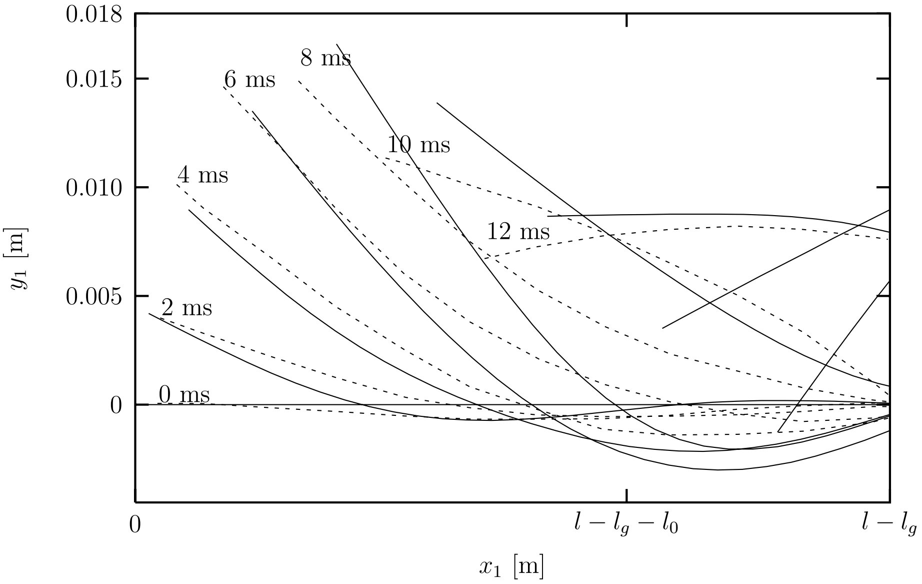

Figure 8: Curves of arrow's deflection on the basis of experimental data, dashed line (- - -) after Pękalski (1990), and on the basis of mathematical model, solid line (——) every 2 ms after release. Bow is modeled as a linear spring where drawing force E_n is proportional to actual drawing length (Fig. 7). The shape of the arrow is shown in a coordinate system fixed to the bow; the arrow in fully draw situation, in the median plane of the bow, is on the horizontal axis and the place of the grip of the bow is point $(l-l_g, 0)$.

both models of the bow is shown in Fig. 11. Let t_g denote the instant that the arrow nock passes the grip. It is important that the nock clears the grip at this moment. After the arrow leaves the string $t_1 = t_l$ it passes the median plane of the bow ($y_1 = 0$) but before $t_1 = t_g$. This is related to the 'Archer's Paradox', (Klopsteg, 1992). The arrow does not slap with its rear end against the grip but snakes around it and this makes the process central to the shot for this movement improves the accuracy of the shot. Pękalski did not consider this important feature of the arrow's kinematics.

Discussion and conclusions

Model validity

Due to the inertia for the bow limbs the SFD and the DFD curves differ significantly (Fig. 7). In this paper we showed that it is better not to simplify the modeling of the dynamic action of a bow–arrow combination by the use of a simple linear spring model for the bow as is done by Pękalski (1987;1990) for the calculation of the acceleration force.

Comparison of the calculated shapes in Figs 1 and 8 shows that in the new model the bending of the arrow is larger. This is obviously caused by the initially rather large normal force associated with the acceleration force, which is neglected by Pękalski (1987;1990).

Our results are in agreement with the experimental results of Gallozzi *et al.* (1987) and

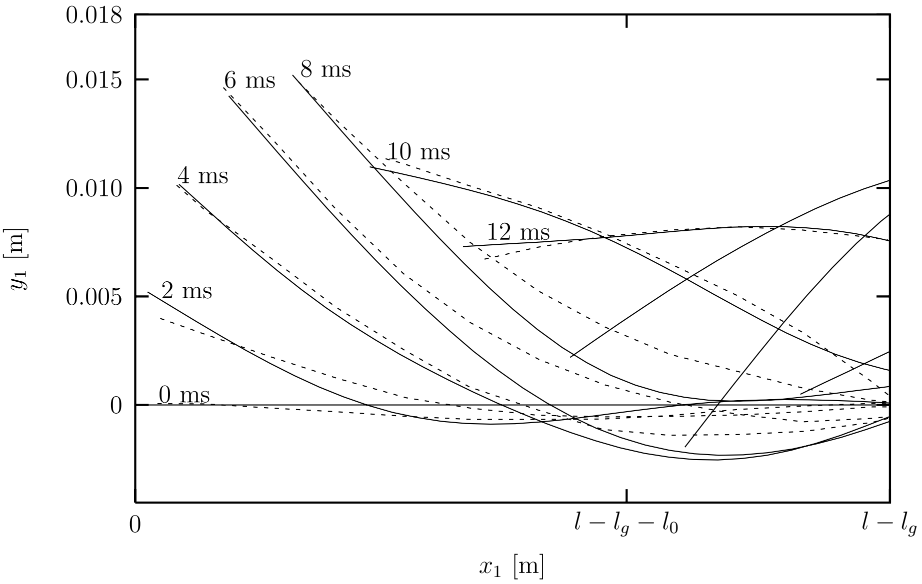

Figure 9: Curves of arrow's deflection on basis of experimental data, dashed line (- - -) after (Pękalski, 1990), and on basis of new mathematical model for the arrow, solid line (——) every 2 ms after release and for model results until arrow nock passes grip. Acceleration force E is predicted force shown in Fig. 7.

Leonardi *et al.* (1987) with respect to the period of contact between the arrow and the grip. The contact ends before the arrow leaves the string (Fig. 10).

In Pękalski (1990) the verification of the mathematical model was performed by comparing two descriptions of the arrow's movement

- Film data of the real archer bow–arrow system.
- A description resulting from the computer simulation.

Comparison of the data from Figs 1 and 9 suggest that the simulation results obtained with the new model fit the experimental data from a high-speed film better on the bounds of the observed intervals of time ($t = 0$ and $t = t_l$) as well as space variables. This supports Pękalski's statement that the influence of the rest's edge elasticity should be taken into consideration partly; also other improvements of the model were responsible for the satisfying fit which verifies the mathematical model.

Model use in sport of archery

For the standard arrow, $d = 17/64$ inch, Pękalski's model predicts that the displacement of the nock of the arrow out of the median plane is zero for a relatively long time interval preceding arrow exit at $t_1 = t_l$, Pękalski (1990, Fig. 8B). His calculations for soft $d = 15/64$ inch and stiff $d = 21/64$ inch arrows suggest that there was no such period for these two arrows. On the basis of these results Pękalski formulated the following definition of a well selected bow–arrow system:

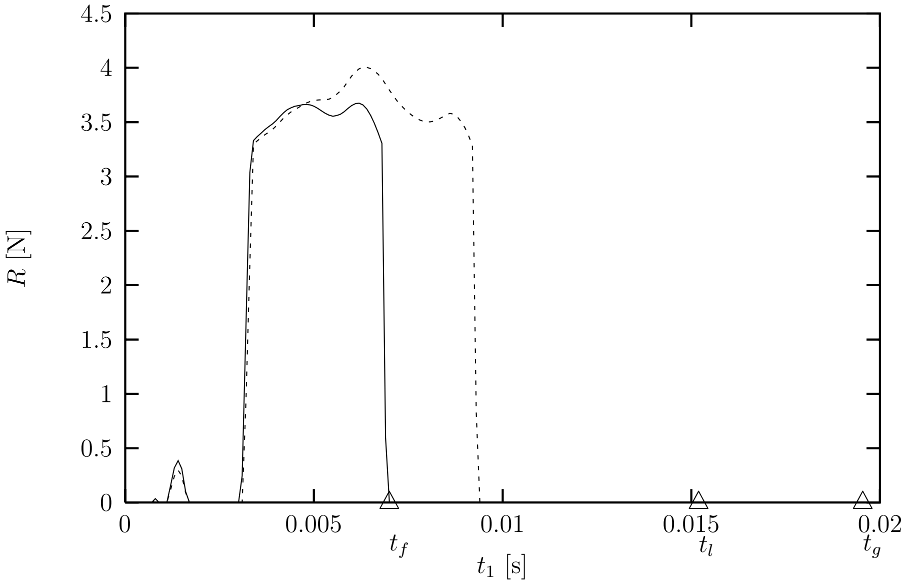

Figure 10: Contact force $R(t_1)$ as a function of time t_1. Solid line (▬▬) is obtained when acceleration force E is predicted force shown in Fig. 7. Arrow is free from grip for a small time period before contact force becomes rather large and leaves grip definitely at time t_f, indicated by the symbol '△' on the t_1-axis. At t_l the arrow leaves the string and at t_g the arrow nock passes the grip. Dashed line (- - -) is obtained when bow is modeled as a linear spring where drawing force E_n is proportional to actual drawing length.

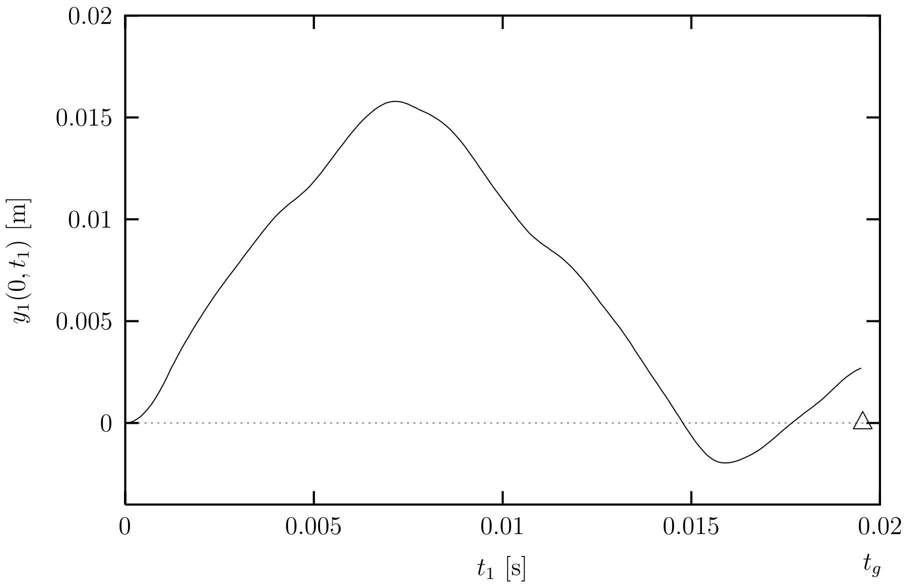

Figure 11: Path of arrow nock for the standard arrow Easton X7 1714 (d = 17/64 inch and g = 14/1000 inch). The acceleration force E is shown in Fig. 7. At t_g, indicated by the symbol '△' on the t_1-axis, the arrow nock passes the grip.

> A well selected bow–arrow sub-system is any system for which the dimensionless parameters of the mathematical model of the arrow's movement during its contact with the bow, have the same values as for the 'standard' system. This means that all well-selected bow–arrow sub-systems should have, after proper rescaling of the axes of the coordinate system and time $(x/l, y/b, t/\theta)$, movement that is identical to the standard arrow.

Thus, his definition tries to formulate in words that the jump in the transverse force acting upon the nock at arrow exit is small for a well chosen arrow. Our results shown in Fig. 11, indicate that the arrow leaves the string approximately at the instant the nock passes the median plane again. Hence, the definition of Pękalski of a well selected bow–arrow system may still be useful.

Archers select the arrow depending on the bow weight and draw length according to recommendations (spine charts) of the manufacturer of the arrow. During the tuning procedure, see (Baier *et al.*, 1976), adjustments to the bow and arrow are made that produce the best possible performance of the arrow—bow—archer combination. Several factors influence the performance, such as the arrow dimensions, types of release, types of 'grip of the bow' and bow parameters, such as the brace height, draw length, and so on. The model proposed by Kooi (1991) for the bow and in this paper for the arrow flight, make it possible to do tuning by simulation on the computer. In these models all parameters have a clear mechanistic interpretation. Therefore arrow clearance can be predicted.

There is, however, a reverse of the medal; more elaborate experiments have to be performed for each individual archer in order to measure how he/she releases the arrow over the finger tips and how he/she moves his/her bowhand during the release phase. One way to proceed is to define a 'standard release' based on experimental data obtained from the analysis of a high speed film of a top archer-competitor. Sensitivity analysis for this standard case would yield the most important parameters.

Nowadays the video camera is already used as a tool for analyzing errors and teaching proper techniques (Bolnick *et al.*, 1993, page 112) and (Sanchez, 1989; Rabska and van Otteren, 1991). For coaching an individual archer-competitor, images from high speed video-recording or photography (for instance 7000 frames per second) taken from above the archer have to be digitized. Experiments similar to those performed in (Keast and Elliott, 1990; Leroyer *et al.*, 1993; Stuart and Atha, 1990) should be performed whereby the archery performance is correlated with movements of the extending fingers during release and of the bowhand during the period the arrow is launched from the bow; the most critical period of the shot. In (Rabska and van Otteren, 1991) a specification for slow-motion video production is given. The extracted data form the input for a computer program based on the work presented in this study. the program runs on a low-costs personal computer. Results from runs with slightly different parameter settings may give insight into the cause of errors and how to improve the technique.

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References

Axford, R. (1995). *Archery Anatomy*. London: Souvenir Press.

Baier, P., Bowers, J., Fowkes, C. R. and Schoch, S. (1976). *Instructor's Manual*. Colorado Springs CO: The National Archery Association of the United States, second edition.

Bolnick, H., Bryant, R., Horn, M., Phillips, R., Rabska, D., Williford, H. and Wilson, L. (1993). *Archery: Instruction Manual*. Colorado Springs CO: National Archery Association of the United States, fourth edition.

Gallozzi, C., Leonardi, L. M., Pace, A. and Caselli, G. (1987). A new method to measure lateral bow accelerations during shooting in archery. In *Biomechanics: Basic and Applied Research* (edited by G. Bergmann, R. Kölbel, and A. Rohlmann), pp. 639–644, Dordrecht, The Netherlands: Nijhoff.

Keast, D. and Elliott, B. (1990). Fine body movements and the cardiac cycle in archery. *J. of Sports Sciences*, **8**, 203–213.

Klopsteg, P. E. (1992). Physics of bows and arrows. In *The Physics of Sports* (edited by A. Armenti), pp. 9–26. Woodbury NY: AIP Press.

Kooi, B. W. (1991). On the mechanics of the modern working-recurve bow. *Computational Mechanics*, **8**, 291–304.

Kooi, B. W. and Sparenberg, J. A. (1997). On the mechanics of the arrow: Archer's Paradox. *Journal of Engineering Mathematics*, **31**(4), 285–306.

Leonardi, L. M., Gallozzi, C., Pace, A. and Dal Monte, A. (1987). Reduction of lateral bow displacement using different torque flight compensators and stabilizers in archery. In *Biomechanics: Basic and Applied Research* (edited by G. Bergmann, R. Kölbel, and A. Rohlmann), pp. 633–638, Dordrecht, The Netherlands: Nijhoff.

Leroyer, P., van Hoecke, J. and Helal, J. N. (1993). Biomechanical study of the final push–pull in archery. *J. of Sports Sciences*, **11**, 63–69.

Mitchell, A. R. and Griffiths, D. F. (1980). *The Finite Difference Method in Partial Differential Equations*. New York: Wiley.

Pękalski, R. (1987). *Modelling and simulation research of the competitor—bow—arrow system (in Polish)*. PhD thesis, Academy of Physical Education, Warsaw.

Pękalski, R. (1990). Experimental and theoretical research in archery. *J. of Sports Sciences*, **8**, 259–279.

Rabska, D. and van Otteren, T. (1991). Easton Technical Video Bulletin #1 – Compound Bow series. Available from Easton Inc., Salt Lake City, Utah, USA. Video.

Sanchez, G. (1989). The winning edge: Tips on archery. Available from Easton Inc., Salt Lake City, Utah, USA. Video.

Stuart, J. and Atha, J. (1990). Postural consistency in skilled archers. *J. of Sports Sciences*, **8**, 223–234.

Timoshenko, S., Young, D. H. and Weaver, W. (1974). *Vibration problems in Engineering.* New York: Wiley.

Tuijn, C. and Kooi, B. W. (1992). The measurement of arrow velocities and the efficiency of working-recurve bows. *European Journal of Physics*, **13**, 127–134.