The Archer's Paradox and Modelling, a Review

[1] B.W. Kooi
Faculty of Biology, Free University, De Boelelaan 1087,
1081 HV Amsterdam, The Netherlands

Abstract

In this paper I review physical and mathematical models dealing with the interaction between the bow and the arrow, especially the 'Archer's Paradox' that is, how the arrow can follow a straight path to its target despite being released outside the median plane of the bow. High-speed photographs and films are used for analysis of the discharge of an arrow from the bow. In mathematical models the arrow is modelled as a slender beam vibrating in a plane perpendicular to the median plane of the bow. The main objective is to predict the natural frequency of the arrow, given its geometric dimensions and material constants as well as the interactions with the bow at the middle of the string and the arrowpass. Natural frequency is important in the evaluation of the matching of the bow-arrow combination. It should be related to the time it takes for the bow to accelerate the arrow from full draw to arrow exit. Another criterion is the requirement that the arrow does not buckle due to the acceleration force exerted on it by the bow. Both relationships have been used to estimate the weight of bows used in the past in those cases in which arrows alone survive.

[1] *History of Technology*, 1998, 20:125-137

1 Introduction

One of the most fascinating phenomena in archery is found in the 'Archer's Paradox'. It would seem that the arrow should fly far left of its target, since it passes on the left side of the bow (in the case of a right-handed bowmen), in the time the string (and therefore the rear-end of the arrow) moves in the median plane of the bow from full draw to the braced position. The 'Archer's Paradox' consists in the fact that the arrow does fly to its mark instead of along a line represented by its axis in braced position. The paradox is illustrated in Figure 1 (taken from Klopsteg[1]). A rigid arrow changes direction when the string is gradually let down. How is it possible for the arrow to pass the bow on its flight to the target without its striking against the grip during its discharge?

Elmer writes that the word *paradox* in this connection was not used until it appeared in the title and text of an article by E.J. Rendtroff in his, "The Toxophilist's Paradox" *Forest and Stream* (1913), February 8.[2] According to Klopsteg, Horace Ford was the first writer of a treatise on the subject to devote some attention to the manner in which the arrow passes the bow.[3] Klopsteg continues:

> The difficulty of the problem presented in the paradox—and by problem I mean the possibility of predicting the line of flight of an arrow, knowing the line of aim, weight of bow, stiffness, weight, and other characteristics of the arrow—is to be found in several causes. The first bears repetition, because of its importance: for any given bow and arrow we are dealing with a large force, acting on a non-rigid body, and varying in magnitude and direction during its entire short period of action. The other causes of difficulty are the large number of factors that affect the result. The fact that arrows can be selected to fly consistently, and behave properly, even when used with different bows, gives some hope that certain simple, fundamental relationships may be discovered which will help to an understanding of the paradox.
>
> The first part of the problem is to ascertain exactly how the arrow gets by the bow. This must be done before we can say why it behaves as it does. In this case, high speed photography gives us the desired information.[4]

Thereafter he describes his experimental set-up for photographing the arrow, and shows a series of pictures taken of the bow being shot by hand. Figure 2 gives a schematic representation of the phases of the arrow in its passage from the bow, based on evidence from speed-flash photography[5]. Hickman predicted that the manner in which the arrow bends and vibrates as it leaves the bow would explain the 'Archer's Paradox'. In order to prove this and to study it further, he decided to try to take pictures of an arrow in flight. In the forties he carried out experiments using high-speed filming. Schumm writes:

> The first high-speed archery pictures were taken for Hickman in 1930 by photographer Harry Day on the roof of the Fiske Building in New York, using a 16mm camera with the speed at 2000 frames per second. Even though the lighting was poor and the distortion due to the optical system was great, the

> information obtained from the pictures was invaluable. For the first time Hickman had some visual proof to back up his suspicions.[6]

In 1938 Hickman used a 4000 frame-per-second camera (which he helped design) to photograph the behaviour of different types of arrows when shot by expert archers. These films revealed that the flexural rigidity (the bending properties) of the arrow is very important.[7] The arrow actually bends around the bow and keeps oscillating in its flight toward the target, as shown in Figure 2.

The oscillation period of the arrow must be matched to the time taken by the arrow to be accelerated past the bowstave, so there is a link between the natural frequency of the arrow and the draw weight and draw length of the bow. This fact has been used to assess the strength of bows used in the past when arrows dating from the times in question were available.[8]

The natural frequency depends on various parameters of the arrow, namely dimensions, namely length and diameter, on material constants, namely specific mass and flexural rigidity (a measure for the change of the curvature caused by loading) as well as on the way the arrow is supported. For instance the natural frequency of the arrow in its free flight toward the target differs from the natural frequency obtained if it were hinged at its two ends, that is to say, the ends act as pivot points and the beam can rotate freely around these points. During discharge, the arrow is supported at two points where it contacts the bow, namely the middle of the string and the arrowpass. The last point changes with time, and the arrow may lose contact before arrow exit, that is when it leaves the string. Moreover, these points move laterally.

These lateral movements of both the arrow nock (the rear-end of the arrow provided with a groove in which the string slightly sticks when the arrow is set in it) and the contact point with the arrowpass (the place on the grip where the arrow passes the bow) are unknown and hard to assess because they depend on the archer's technique of release. Hodgson writes

> In the first place I have been able to demonstrate that our paradox contains or is accompanied by several minor and similarly freakish actions contrary to what we might expect.[9]

Klopsteg was also aware of the fact that the paradox carries within itself an intricate problem:

> It was soon realized that it is very desirable to know the path of the string, as well as of the bow hand, with the expectation that this might help to explain the paradox.[10]

Klopsteg continues:

> Mr. Nagler believes that the initial bending of the arrow as shown in the pictures is caused by the leftward motion of the string as it comes off the drawing fingers. This may have some effect in producing the bend. A sudden force to the left

> at the nock of the arrow would, because of the inertia of the arrow, cause the latter to exert a force against the arrow plate [A piece of hard material set on the arrowpass to take the chafe of the shaft of the arrow] and thus produce the kind of bending observed. At the same time, the fact that the force exerted by the string is not acting in the longitudinal axis of the arrow but to the right of it will also cause bending of the same kind. It is probable that both the causes are effective in producing the initial bending of the arrow which is so clearly shown in the pictures.[11]

Nagler and Klopsteg take the case of a right-handed archer using the *Mediterranean release*. When the bow is fully drawn, it is kept in this position by three fingers of the archer hooked on the string, the forefinger above and two fingers below the nock (grooved rear end) of the arrow. Right-handed archers draw the bow with the right hand and hold the bow in their left hand. Observe that in former times the arrow rested on the knuckle of the forefinger of the bowhand. Hence, the side on which the arrow passes the bow differs for right- and left-handed archers.

Morse distinguishes five different type of releases.[12] They are: primary, secondary, tertiary, Mediterranean and Mongolian release. Kroeber deals with the geographical distribution of these releases.[13]

In Asian countries the arrow was shot through the Mongolian release or thumb-release. In this case the string slips over the thumb and hence the distortion out of the median plane of the bow is in the opposite direction to that of the Mediterranean release. Consequently the arrow is placed on the thumb of the bowhand-side. McEwen reports that:

> The Mongolians do not always place the arrow on the right side of the bow (for a right handed archer). Fully as many placed the arrow on the left as the right despite using the thumb draw. It has always been stated that the arrow must be on the right of the bow when drawing with the right thumb but the Mongolians seemed able to shoot just as accurately whichever method was employed. When I asked why the arrow was put on the left side of the bow and not the right, the answer I got was usually, “I have always done it this way”. I have puzzled endlessly about this and have concluded that with the correct spine of the arrow for the method used, it makes no real difference and accuracy can be achieved. A different value of spine is needed according to which side of the bow the arrow is placed.[14]

For similar reasons, namely that primitive archers should to some extent be able to compensate for non optimal matched vibration frequencies, Blyth proposed a structural stability criterion to assess the strength of bows based on the dimensions of contemporary arrows.[15] The criterion is that the arrow will not collapse under its own inertia during acceleration.

Hickman invented the so-called centre-shot bow. All modern bows are of this type. A cut-out of the rigid middle part of the bow, called the grip, handle or riser, allows the arrow to pass through the median plane of the bow. The arrow is supported by an arrow rest, often a button (shock absorber) with a built-in spring.

Curiously, it has been reported by Quayle that arrows fired straight ahead do not need to be matched to their bows.[16] The arrows were restrained in the left/right and the up/down direction so as to centre fire. Shot by experts using pinch type releases (pinch grip on the nock between the thumb and first finger and thus without a movement out of the median plane) or their equivalent in a mechanical release device, arrows of vastly different spine (stiffness, see below) all fired straight ahead.

High-speed films made by Pękalski show, however, that there is still a vibratory movement of the arrow in modern target shooting. During the release the string still slips over the finger tips out of the median plane.[17]

In the next two sections I describe physical and mathematical models which have been developed to find out exactly what happens during the launch of the arrow. Physical models (mechanical shooting machines - which give reproducibility) together with photography have been used to study the 'Archer's Paradox'. The experiments conducted showed that the flexibility of the arrow has to be matched to the weight of the bow, and mathematical models are used to quantify the relationship. This relationship will be used to estimate the weight of bows compatible with the Westminster Abbey arrow discussed in Pratt and Hardy.[18] Blyth used a structural stability argument to estimate the weight of bows used with the Egyptian reed arrow.[19] I shall also make use of a vibrating beam model proposed by Pratt[20] and the model proposed by Pękalski.[21]

2 Physical models

Elmer gives a summary of Dr. Rendtroff's article which mentions the shooting machine Elmer experimented with as early as 1912. Elmer believes he was the first in America and writes:

> The idea of a shooting-machine was independently conceived by me, but there is a comment in the *Archer's Register* for 1886 on something of the sort used in England before that time. The bow was fastened in a "machine" for testing results. [· · ·] His [the author William Ford's] experiments convinced him that the arrow would fly to the left if the bow held rigid and that its flight in a hand-held bow was possible because it shoved the bow aside. [· · ·] Probably the test is as old as the cross-bow and the great projectile-throwing engines of antiquity.[22]

A number of articles collected in Hickman et al., namely those of Hodgson and Klopsteg deal with the 'Archer's Paradox'.[23] These articles describe experiments with a shooting machine and also hand shooting; high speed spark photography and filming is also described. Experiments were performed in which subsequently one parameter was changed to find the sensitivity with respect to the changed parameter.

To study the movements of the middle of the string (the rear end of the arrow) and the bowhand (the arrowpass), Klopsteg wired a bow so that sparks were produced simultaneously at the two locations.[24] Cameras recorded a succession of sparks. Later, Klopsteg fastened a light stylus to the bow just above the arrow, and let the stylus trace the motion of the bow on a piece of suitable recording paper while the bow was shot. Hodgson discusses also the path of the string, the movement of the bow at the hand and at the tips, the path of the arrow as it bends around the bow and the antics of the arrow as it leaves the bow and gets away on its flight. He states:

> In shooting an arrow we have to deal with, first, the lateral impulse imparted to the string and arrow by the releasing fingers, second, with the acute angle of the arrow and the path of the empty string, also with the center of gravity of the arrow.[25]

Pękalski describes experiments with a shooting machine called the DMLA, an acronym formed from the words 'Device for Mechanical Loosing of an Arrow'.[26] He used it to measure the sensitivity of the shooting accuracy with respect to the torsion of a bow around the vertical axis through the arrow rest. The shooting distance was 30 m. Two different sensitivities were found, namely -15.8 cm/degree for negative torsion and 21.9 cm/degree for positive torsion. This observed asymmetry is, according to Pękalski

> [$\cdots$] connected with the influence of the torsion on the transverse vibrations of an arrow in the horizontal plane. With the sign change the direction of the first deflection of the arrow changes, thus changing the interaction between the arrow and the nocking point.[27]

Besides the arrow released from the DMLA Pękalski also filmed (1000 Hz high-speed 16-mm cine film) the arrow release by a member of the Polish National team. Figure 3 shows the arrow's movement taken from the film made with the camera placed vertically over the archer. Pękalski writes:

> The films showed that there were no qualitative differences in the movement of the arrow released using the two methods.[28]

Klopsteg states:

> Although the shooting machine fulfills a very useful purpose in studies of the bow and arrow and in testing of arrows, it seems doubtful whether a machine can be constructed which exactly reproduces hand shooting.[29]

However, Hickman designed a mechanical shooting device, which was built as far as can be determined in 1927, which

> [$\cdots$] was constructed to duplicate human movement as much as possible. The bow was clamped onto an arm that pivoted, and the device which held the string was locked into position with a trigger. In order to assure the smoothest release possible the trigger was released pneumatically by squeezing a bulb, similar to a camera action.[30]

3 Mathematical models

In this paper a linguistic description of the mathematical models is given and the results of the calculations are presented graphically. The mathematical models are described in their entirety elsewhere.[31]

In archery the flexural rigidity (a quantity defined in structural engineering) of the arrow is represented as the *spine*. Rheingans and Nagler discuss the spine and arrow design from the modelling point of view.[32] To measure the spine of an arrow, for instance one which is 29 inches long, a bob weighing 1.94 lb is hung in the centre of the arrow supported at two points separated a distance (the *span*) of one inch less than the arrow length, in our example 28 inches. The deflection measured in inches is the *spine* of the arrow. This is the definition of Easton, manufacturer of arrow shafts. The following relationship holds:

$$spine = \frac{load \times span^3}{48 flexural\ \ rigidity} \tag{1}$$

In this formula *spine* and *span* are measured in m, *load* in N, *flexural rigidity* in $\mathrm{N\,m^2}$. Rheingans and Nagler propose to take the deflection in inches under 2 lb load in the middle on a 26 inches span as a standard for comparison.[33] In the GNAS (Grand National Archery Society), definition the load is 1.5 lb with the arrow supported at the base of the nock and at the shoulder of the tip, and the deflection is measured in hundredths of an inch. Please see endnote[34] for explanations of the units of measurement used here and in the following.

3.1 Structural strength

Liston discusses the structural strength of the arrow. The accelerations involved in the propulsion of the arrow are enormous; for a 50 lbs bow and an arrow with mass 0.02 kg the acceleration after release is about 12500 $\mathrm{m/s^2}$.[35] The ability of a section of the shaft of the arrow to resist the in-line compression force is nonetheless massive and need not be analysed.

3.2 Structural stability model

Liston distinguishes two ways of buckling; a slender column buckling firstly due to the mass of the tip, and secondly under its own mass. Blyth states Euler's reason for the buckling. Here both ends are hinged. One-half of the mass of the shaft is located at the tip, the mass of the nock not being taken into account.[36] This yoelds the maximum load supplied by the string, which is taken to be equal to the draw weight of the bow. The latter model predicts that the modern arrow will buckle. This agrees with the experience of Liston:

> The author tried nocking an arrow and gradually pushing against a block. When the bow was about half way drawn it became obvious that the arrow was becoming unstable and would break if the experiment were continued.[37]

3.3 Vibrating beam model

Pratt used the well-known equation for an approximation of the natural vibration frequency of a vibrating beam.[38] He uses a very simple model for the bow. The acceleration of the arrow is assumed to be constant and equal to half the weight of the bow. Pratt states that the time for the rear-end of the arrow to travel a distance equal to the draw of the bow, has to be equal to the time comprising one and a quarter vibrations of the arrow, that is, 1.25 times the natural vibration period of the arrow.

3.4 Pękalski's model

Pękalski models the arrow as an elastic beam vibrating in a plane perpendicular to the median plane of the bow and through its line of symmetry.[39] Boundary conditions are prescribed at the arrow head and at the arrow nock. The bow is modelled as a simple linear spring with 75% efficiency for the longitudinal direction as well as the transverse direction. The elasticity of the bow in the latter case drives the middle of the string, where it sticks in the arrow nock, toward the median plane of the bow. Initially the arrow has contact with the bow at the grip and the nock is situated out of the median plane.

Figure 3 shows the experimentally observed curves of the arrow's deflection and the curves on the basis of Pękalski's mathematical model every 2 milliseconds (ms) after release.

4 Two ancient arrows

According to Nagler and Rheingans:'the weight of the bow should be proportional to the flexural rigidity of the arrow but the mass of the arrow cancels out'.[40] The constant of proportionality equals 55.03 m^{-2}, but no derivation of this value is given. The heavier the arrow, the lower its velocity, and similarly its vibrating time, both in the same power; therefore, the deflection of the arrow should be constant for any particular bow but should be decreased for heavier bows and increased for lighter bows. Rheingans and Nagler give a table which contains a very fair sample of how the deflection (spine) of the arrow should depend on the weight of the bow. Pratt and Hardy presented these data graphically and it turned out that the reciprocal weight was linear in the reciprocal spine.[41] They extrapolated the data of Nagler and Rheingans to estimate the weight of the bow suitable for an arrow found in Westminster Abbey discussed in Pratt and Hardy.[42] The arrow was found in 1878 lodged in one of the turrets of Henry V's Chantry. The flexural rigidity of the arrow is 12.82 $\mathrm{N\,m}^2$ (0.164 inches deflection under a 2 lb load on a 26 inches span).

All criteria derived from the models discussed in the preceding section lead to the same relationship, *the weight of the bow is proportional to the flexural rigidity of the arrow*, given only that the constant of proportionality depends on the dimensions of the arrow and the bow.

Figure 4 gives the weight as a function of the flexural rigidity for a number of these criteria. When the efficiency is 67% (this value was calculated for a Mary Rose bow A812 with weight 110 lb at 30 inches draw and for a 48 g arrow)[43] the vibrating beam model proposed by Pratt gives 683 N (153 lb) and this almost equals the value given by Pratt and Hardy according to the Nagler and Rheingans' criterion, the latter being 705 N (158 lb) and Pękalski's model prediction being 755 N (170 lb). The structural stability model proposed by Blyth yields a much lower value, namely 381 N (86 lb) instead of 705 N (158 lb).[44]

In Figure 5 the results for a reed arrow mentioned by Blyth are shown.[45] Blyth measured a number of reed arrows in the Pitt Rivers Museum at Oxford, which were found together with an angular bow in an Egyptian tomb of the 26th Dynasty (the 7th Century B.C.). The flexural rigidity of the arrows is taken to be 0.986 $\mathrm{N\,m^2}$ (160 GNAS). The vibrating beam model proposed by Pratt (assuming that the efficiency is 43% - a value mentioned by Blyth) gives 47.7 N (10.7 lb). The structural stability model proposed by Blyth gives 29.7 N (6.7 lb), which is remarkably low.

5 Discussion and conclusions

In the literature it is recognized that the 'Archer's Paradox' constitutes an intricate problem. The flexible arrow snakes around the bow, but the movement of the bowhand and the archer's technique of release are also important. These latter facts restrict the usefulness of shooting machines. Shooting machines are, however, very useful devices because reproducible shots can be produced and this facilitates sensitivity studies.

The arrow is modelled as a bending beam and its lateral movements are determined by complex boundary conditions at the point of contact with the string and at the arrowpass. The contact of the arrow with the arrowpass leads to a moving boundary condition. This means that the position on the arrow of this point of contact changes with time. The contact lasts for a restricted time interval and the arrow is free from the arrowpass before the arrow leaves the string. The string is pushed out of the median plane to the left during the release and the archer's bowhand moves to the right during the acceleration of the arrow.

These movements of the bowhand and the string during release out of the median plane, should be taken into account in the mathematical modelling. This is done in Kooi and Sparenberg.[46] Such a mathematical model can be used to evaluate the criteria for matching the bow and arrow more accurately. It gives more detailed results, not only qualitatively, but quantitatively as well, on the movements of the arrow during its launch and thereby on the 'Archer's Paradox'. However, with application of such a model more detailed information needed. With research on ancient arrows this additional information is missing. This lack hampers the use of this model for the estimation of the weight of ancient bows of coeval arrows.

The results obtained with the models presented confirm the results given in Pratt and Hardy, and Blyth for the two ancient arrows, the Westminster Abbey arrow and the Egyptian reed arrow, respectively.[47] The buckling criterion predicts the lowest bow weight that seems unrealistically low. Evaluation of modern arrows shows that these buckle under the draw weight of the bow, indicating that the structural stability model seems to be too conservative. Liston remarks that the launch is completed before the arrow has a chance to buckle to the point of permanent deformation.[48] While it is commendable to avoid instability in most areas of engineering in the case of arrows, however, this is questionable.

It is of interest to quote Rees where he states that the vibrating beam model

> ... suggested that the heavy 60-gram war arrow as used at Agincourt could have been shot from bows with draw weights of over 450 N, but this seemed an unreasonably large value; until about 1980 a figure closer to 350 N was thought more likely. The high values were confirmed by the study of over 100 longbows and 3000 arrows recovered from the Mary Rose.[49]

References

1. C. N. Hickman, F. Nagler, and P. E. Klopsteg, *Archery: the technical side* (National Field Archery Association, Redlands, Ca, 1947), 178.
2. R. P. Elmer, *Target Archery* (Hutchinson's Library of Sports and Pastimes, London, 1952).
3. C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 134.
4. C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 135. The weight of bow is defined as the force exerted by the archer in the middle of the string and on the grip of the bow in fully drawn position. Notice that the bow weight does not pertain to the actual weight caused by gravity. The weight of the arrow is the actual weight caused by gravity. In all formulations I use the mass of the arrow.
5. C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 182.
6. M. M. Schumm, *Clarence N. Hickman the Father of Scientific Archery* (Minisink Hills, Pa, 1990), 86-87.
7. R. P. Elmer, *op. cit.* (2), 352.
8. P. L. Pratt and R. Hardy, 'The arrow found in Westminster Abbey in 1878', *Journal of the Society of Archer-Antiquaries*, 1975, 18: 22–24; R. Hardy, *Longbow* (Patrick Stephens, Sparkford, Nr Yeovil, Somerset, 1992); G. Rees, 'The longbow's deadly secrets', *New Scientist*, 1993, June 5, 24–25.
9. C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 258.
10. C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 136.
11. C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 142.
12. E. S. Morse, 'Ancient and modern methods of arrow-release', *Bulletin of the Essex Institute*, 1885, 17: 145–198; E. S. Morse, *Additional notes on arrow release* (Peabody Museum, Salem, Massachusetts, 1922).
13. A. L. Kroeber, 'Arrow release distributions', *Univ. of California publications in American Archaeology and Ethnology*, 1927, 23: 283–96.
14. E. McEwen, 'Mongolian journey', *J. S. A-A.*, 1993, 36: 19–30, p. 22.
15. P. H. Blyth, 'Ballistic properties in ancient Egyptian arrows', *J. S. A-A.*, 1980, 23: 34–39.
16. C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 211–15.

17. R. Pękalski,
Modelling and simulation research of the competitor—bow—arrow system (in Polish) (PhD thesis, Academy of Physical Education, Warsaw, 1987); R. Pękalski, 'Experimental and theoretical research in archery', *Journal of Sports Sciences*, 1990, 8: 259–79.

18. P. L. Pratt and R. Hardy, *op. cit.* (8).

19. P. H. Blyth, *op. cit.* (15).

20. R. Hardy, *op. cit.* (8), 226–32.

21. R. Pękalski, 'Experimental and theoretical research in archery', *Journal of Sports Sciences*, 1990, 8: 259–79.

22. R. P. Elmer, *op. cit.* (2), 364.

23. C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 258–66; P. E. Klopsteg, 'Physics of bows and arrows', *American Journal Physics*, 1943, 11: 175–192; C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 134–146.

24. C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 136.

25. C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 258.

26. R. Pękalski, *op. cit.* (21).

27. R. Pękalski, *op. cit.* (21).

28. R. Pękalski, *op. cit.* (21).

29. C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 140.

30. M. M. Schumm, *op. cit.* (6), 63.

31. B. W. Kooi, 'Arrow dynamics', *European Journal of Physics* (forthcoming).

32. C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 215–25; C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 226–36; F. Nagler, *Archery, an engineering view* (Frank Taylor & Son, Albany, Oregon, 1940).

33. C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 226–36.

34. In archery the second is used for unit of time, the inch as unit of length, the lb as unit of force and the grain as unit of mass. In this paper I use the system referred to as the SI units. In this system three base units for time, length and mass are second (s), meter (m) and kilogram (kg), respectively. The derived unit of force is the Newton (N). A force of 1 newton (N) will accelerate 1 kilogram (kg) of mass at 1 meter per second per second (m/s^2). Some conversion factors are: 1 inch=0.0254 m, 1 lb=4.448 N and 1 grain=0.0000648 kg. Furthermore I use the standard prefixes to denote multiples of tens, for example 0.001 or 10^{-3} seconds are referred to as 1 millisecond (ms).

35. T. L. Liston, *Physical Laws of Archery* (Liston & Associates, San Jose, Ca, 1988), 9-1.

36. P. H. Blyth, *op. cit.* (15).

37. T. L. Liston, *op. cit.* (35), 9-1.

38. R. Hardy, *op. cit.* (8), 226–32.

39. R. Pękalski, *op. cit.* (21).

40. C. N. Hickman, F. Nagler, and P. E. Klopsteg, *op. cit.* (1), 226–36.

41. P. L. Pratt and R. Hardy, *op. cit.* (8), Figure 3.

42. P. L. Pratt and R. Hardy, *op. cit.* (8).

43. R. Hardy, *op. cit.* (8), 217.

44. P. L. Pratt and R. Hardy, *op. cit.* (8); R. Hardy, *op. cit.* (8), 226–32; P. H. Blyth, *op. cit.* (15); R. Pękalski, *op. cit.* (21).

45. P. H. Blyth, *op. cit.* (15).

46. B. W. Kooi and J. A. Sparenberg, 'On the mechanics of the arrow: Archer's Paradox', *Journal of Engineering Mathematics*, 1997, 31: 285–306.

47. P. L. Pratt and R. Hardy, *op. cit.* (8); P. H. Blyth, *op. cit.* (15).

48. T. L. Liston, *op. cit.* (35), 9-2.

49. G. Rees, *op. cit.* (8).

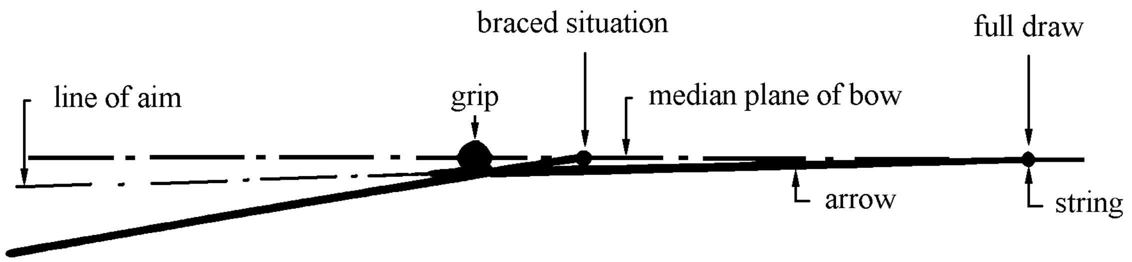

Figure 1: Illustration of the archer's paradox (after Klopsteg[1]). A rigid arrow will change direction as the string gets closer to its equilibrium point in braced position. Therefore it would seem that the arrow should fly far left, since it passes on the left side of the bow.

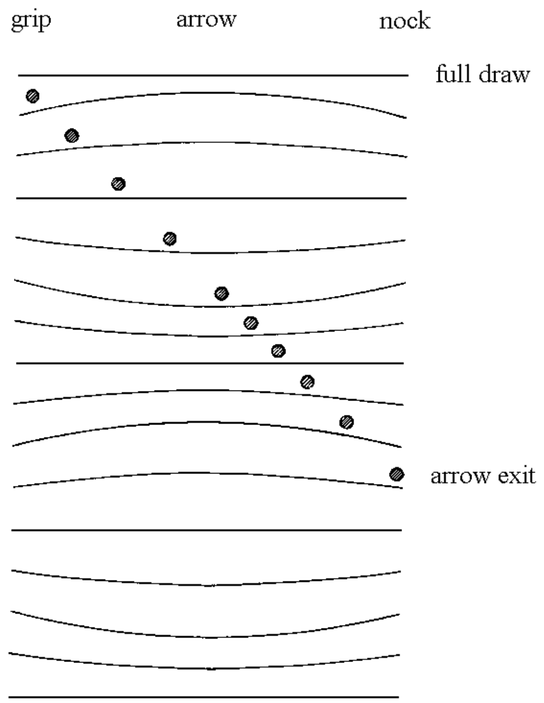

Figure 2: Schematic representation of the phases of the arrow in its passage from the bow, based on evidence from speed-flash photography (after Klopsteg[1]). The shape of the arrow is shown in a coordinate system fixed to the arrow.

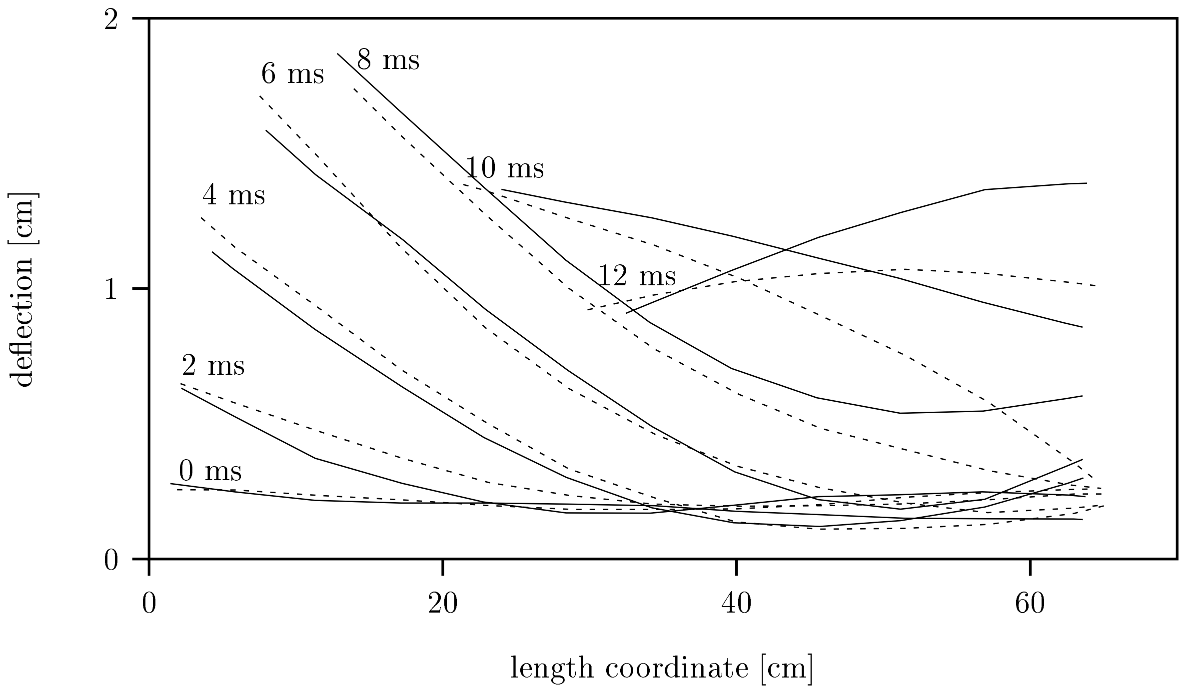

Figure 3: Curves of the measured arrow deflections (- - -) and the arrow deflections predicted by Pękalski's mathematical model (——) every 2 milliseconds (ms) after release (after Pękalski[17]). The shape of the arrow is shown in a coordinate system fixed to the bow; the arrow in the fully draw position is the line at 0 ms and the place of the grip of the bow is close to the bottom right corner.

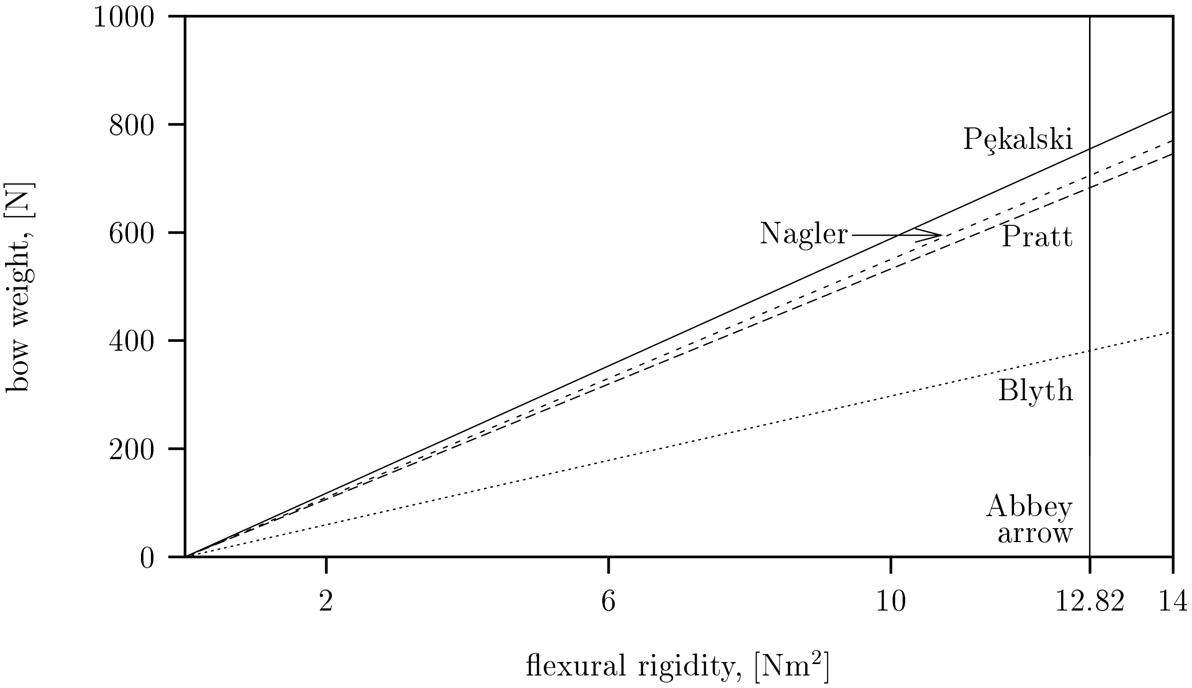

Figure 4: The weight of the bow as function of the flexural rigidity for a number of criteria for the Westminster Abbey arrow discussed by Pratt and Hardy.[8] The curve (- - -) represents the criterion proposed by Nagler and Rheingans and also used by Pratt and Hardy.[39] Curve (——) is based on the criterion proposed by Pękalski.[17] Curve (— —) represents the relationship based on the vibrating beam model proposed by Pratt.[20] Curve (....), represents the criterion proposed by Blyth[15] which follows from the structural stability model.

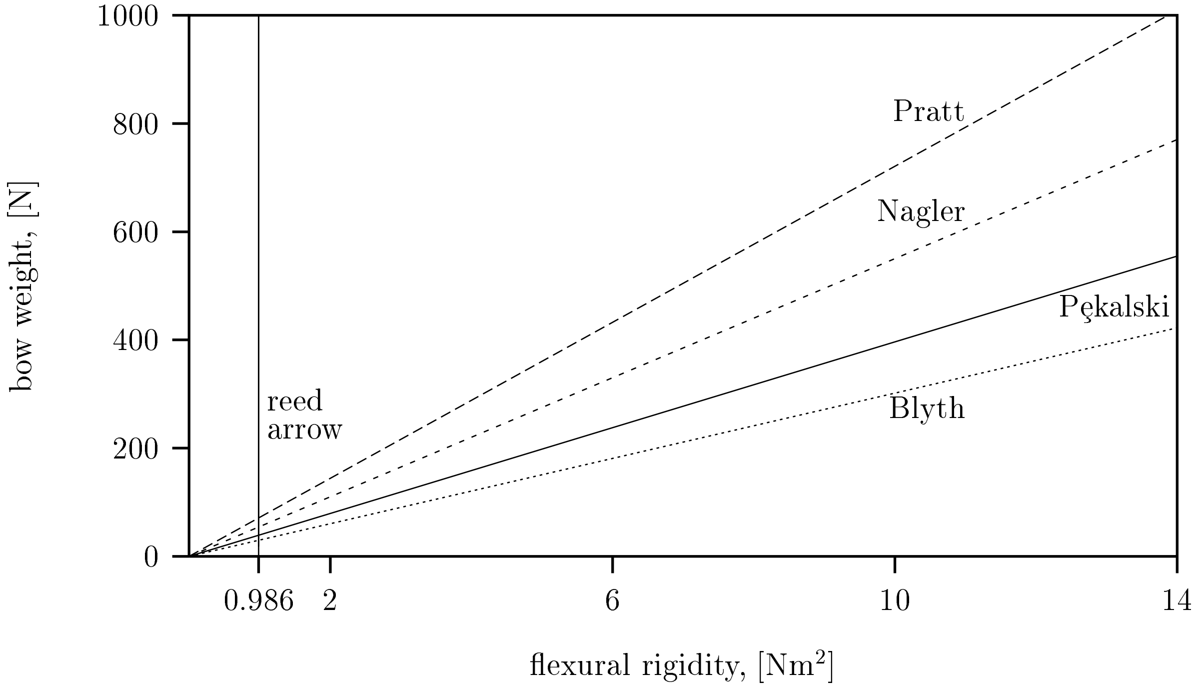

Figure 5: The weight of the bow as function of the flexural rigidity for a number of criteria for the reed arrow discussed by Blyth[15]. Curve (- - -) represents the criterion proposed by Nagler and Rheingans[39]. Curve (——) is based on the criterion proposed by Pękalski.[17] Curve (— —) represents the relationship based on the vibrating beam model proposed by Pratt.[20] Curve (....), represents the criterion proposed by Blyth[15] which follows from the structural stability model.